AP Calculus BC
2000 Student Samples

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Work for problem 2(a)

**Runner A**

\[ \begin{align*}
  \text{at} & = \frac{10 - 0}{3 - 0} = \frac{10}{3} \\
  \text{ds} & = \frac{10}{3} (t - 3) \\
  v(t) & = \frac{10}{3} t \\
  s(2) & = \frac{10}{3} (2) \\
  = & \, 6.667 \text{ m/s} \\
\end{align*} \]

**Runner B**

\[ \begin{align*}
  v(t) & = \frac{24t}{2t+3} \\
  v(2) & = \frac{24(2)}{2(2)+3} \\
  = & \, 4.857 \text{ m/s} \\
\end{align*} \]

Work for problem 2(b)

**Runner A**

\[ \begin{align*}
  v(t) & = \frac{10}{3} t \\
  (t) = v'(t) & = \frac{10}{3} \\
  a(t) & = v'(t) = \frac{10}{3} \\
  s(2) & = 3.333 \text{ m/s}^2 \\
\end{align*} \]

**Runner B**

\[ \begin{align*}
  v(t) & = \frac{24t}{2t+3} \\
  a(t) & = v'(t) = \frac{(2t+3)(24) - (2)(24t)}{(2t+3)^2} \\
  & = \frac{48t + 72 - 48t}{(2t+3)^2} \\
  & = \frac{72}{(2t+3)^2} \\
  a(2) & = \frac{72}{(2(2)+3)^2} \\
  = & \, 1.416 \text{ m/s}^2 \\
\end{align*} \]

Continue problem 2 on page 7.
Work for problem 2(c)

Runner A

Total Distance = \( \int_0^{10} v(t) \, dt \)
\[ = \int_0^3 (\frac{10}{2}t) \, dt + \int_3^{10} (10) \, dt \]
\[ = 15m + 70m \]
\[ = 85 \text{m} \]

Runner B

Total Distance = \( \int_0^{10} v(t) \, dt \)
\[ = \int_0^{10} (\frac{24t}{2t+3}) \, dt \]
\[ = 83.336 \text{m} \]
Work for problem 2(a)

a. Velocity of Runner A = (0,0) (3,10)

\[
\frac{10 - 0}{3 - 0} = \frac{10}{3} = \frac{10}{3} \text{ m/s}
\]

\[
(3,10)(2,4) = \frac{4 - 10}{2 - 3} = \frac{-6}{-1} = 6.
\]

= \boxed{6.67 \text{ m/s}}

---

Work for problem 2(b)

a. Runner b?

\[
y_1 = a
\]

\[
y = V_0 + \frac{g}{2} = 6.67 \text{ m/s}
\]

\[
+ = 25
\]

\[
\frac{6.67}{2} = 3.33 \text{ m/s}^2
\]

---

b. \( v(t) = \frac{24t}{2t + 3} \)

\[
v(2) = \frac{(24)(2)}{2(2) + 3} = \boxed{6.857 \text{ m/s}}
\]

---

a. \( v_0 \) runner b?

\[
\alpha = \gamma
\]

\[
v = 6.857 \text{ m/s}
\]

\[
+ = 28
\]

\[
\frac{6.857}{2} = 3.4285
\]

\[
\boxed{3.4285 \text{ m/s}^2}
\]

Continue problem 2 on page 7.
Work for problem 2(e)

Runner A

\[ d = v + \]

area under curve from 0 to 10

\[ = \frac{1}{2}(3 \cdot 10) + 7(10) \]

85 m

Runner A

Runner B

\[ \int v(t) = d(t) \]

\[ \int_{0}^{10} \frac{24}{2t+3} = \]

83.334 m

Runner B
Work for problem 2(a)

Runner A

\[ v(t) = 7 \text{ m/s} \]

Runner B

\[ v(t) = \frac{24(t)}{2(t)^2 + 3} \]

\[ = \frac{48}{7} \]

\[ = 6.85 \text{ m/s} \]

Work for problem 2(b)

Runner A

\( (2, 7) \) & \( (3, 10) \)

\[ \text{acceleration} \]

\[ a(t) = \frac{10 - 7}{3 - 2} = 3 \text{ m/s}^2 \]

Runner B

\[ v(t) = \frac{24t}{2(t)^2 + 3} \]

\[ a(t) = \frac{(34)(2t^2 + 3) - (24t)(2)}{(2t^2 + 3)^2} \]

\[ = \frac{48t + 72 - 48t}{(2t^2 + 3)^2} \]

\[ a(2) = \frac{72}{(2(2))^2} \]

\[ = \frac{72}{(4)} \]

\[ = 18 \text{ m/s}^2 \]

Continue problem 2 on page 7.
Work for problem 2(c)

Total distance covered by Runner A = area under the v-t graph.

\[ \frac{1}{2} \left( \frac{5}{2} \right) (2) + (7)(10) \]

\[ = 15 + 70 \]

\[ = 85 \text{ m} \]

Total distance covered by Runner B = \( \int_0^{10} v(t) \, dt \)

\[ = \int_0^{10} \frac{24t}{2t+3} \, dt \]

\[ = 83.3 \text{ m} \]