# AP Calculus BC
# 2000 Student Samples

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 A.

Work for problem 3(a)

$$f^{0}(5)=\frac{(-1)^0\, 0!}{2^0(0+2)}=\frac{1}{2}$$

$$f'(5)=\frac{(-1)\cdot 1}{2(3)}=-\frac{1}{6}$$

$$f^{2}(5)=\frac{(-1)^2\, 2!}{4(4)}=\frac{1}{8}$$

$$f^{3}(5)=\frac{(-1)\, 3!}{8(5)}=\frac{-6}{40}=-\frac{3}{20}$$

$$P_3=\frac{1}{2}-\frac{1}{6}\frac{(x-5)}{1!}+\frac{1}{8}\frac{(x-5)^2}{2!}-\frac{3}{20}\frac{(x-5)^3}{3!}$$

$$\boxed{P_3=\frac{1}{2}-\frac{1}{6}(x-5)+\frac{1}{16}(x-5)^2-\frac{1}{40}(x-5)^3}$$

Work for problem 3(b)

$$\sum_{n=0}^{\infty}\frac{(-1)^n \cancel{n!}}{2^n(n+2)}\cdot\frac{(x-5)^n}{\cancel{n!}}$$

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-5)^n}{2^n(n+2)}$$

Ratio Test:

$$\lim_{n\to\infty}\left|\frac{\cancel{(x-5)^n}(x-5)}{\cancel{2^n}\cdot 2(n+3)}\cdot\frac{\cancel{2^n}(n+2)}{\cancel{(x-5)^n}}\right|<1$$

$$\lim_{n\to\infty}\left|\frac{n+2}{2n+3}\cdot(x-5)\right|<1$$

$$\frac{1}{2}|x-5|<1$$

$$|x-5|<2$$

Continue problem 3 on page 9.

radius of convergence = 2

Work for problem 3(c)

Alternating series, so → $R_n \le |a_{n+1}|$ ← Remainder left off is less than first term

(R = error)

$R_6 \le \left| \frac{(6-5)^7}{2^7(7+2)} \right|$

$R_6 \le 8.68055556 \times 10^{-4}$

$R_6 \le .00086805556$

and

$.0008680556 \le .001$

so the sixth degree Taylor polynomial approximates f with an error less than $\frac{1}{1000}$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

$C_1$

Work for problem 3(a)

$$\frac{f^n(5)\cdot(x-5)^n}{n!} = \frac{(-1)^n\cdot(x-5)^n}{2^n(n+2)}$$

$$\frac{1}{2} - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40}$$

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Work for problem 3(b)

ratio test

$$\frac{(-1)^{n+1}\cdot(x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n} = \lim_{n\to\infty}\left|\frac{(x-5)(n+2)}{2(n+3)}\right| = \frac{1}{2}$$

$$\left|\frac{x-5}{2}\right| < 1$$

$$-2 < |x-5| < 2$$

$$\boxed{3 \leq x \leq 7}$$

Continue problem 3 on page 9.

$C_2$

Work for problem 3(c)

6th degree term

$\frac{(x-5)^6}{512} \Rightarrow \frac{1}{512}$

$x=6$

a sixth degree Taylor polynomial will aproximate $f(6)$ with an error less than $1/1000$ because every term after it has a value of less than $1/1000$ for $f(6)$

next term

$\frac{(x-5)^7}{1152} = \frac{1}{1152}$

$x=6$

**END OF PART A OF SECTION II**

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 E₁

Work for problem 3(a)

centered at $x=5$ so $c=5$

$$P(x) = f(c) + f'(c)(x-c) + \frac{f''(c)(x-c)^2}{2!} + \frac{f'''(c)(x-c)^3}{3!}$$

$$P(x) = 5 + \frac{(-1)^{(1)}(1!)}{2^{(1)}(1+2)}(x-c) + \frac{(-1)^{(2)}(2)!}{2^{(2)}((2)+2)} \cdot \frac{(x-c)^2}{2!} + \frac{(-1)^{(3)}(3)!}{2^{(3)}((3)+2)} \frac{(x-c)^3}{3!} + \ldots$$

$$P(x) = 5 - \frac{(x-5)}{6} + \frac{(x-5)^2}{16} - \frac{(x-5)^3}{40} + \ldots$$

$n^{th}$ term given by $\frac{(-1)^n n!}{2^n(n+2)} \cdot \frac{(x-5)^n}{n!} = \frac{(-1)^n (x-5)^n}{2^n(n+2)}$

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Work for problem 3(b)

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x-5)^{n+1}}{2^{n+1}(n+3)} \cdot \frac{2^n(n+2)}{(-1)^n(x-5)^n} \right| = \lim_{n\to\infty} \left| \frac{(x-5)(n+2)}{2(n+3)} \right| = \left|\frac{x-5}{2}\right| \lim_{n\to\infty} \frac{n+2}{n+3} = \left|\frac{x-5}{2}\right|$$

$\left|\frac{x-5}{2}\right| < 1$ by ratio test, if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$, series converges

$|x-5| < 2$

$x-5=2$ or $5-x=2$

$x=7$ or $x=3$

$3 < x < 7$

∴ the radius of convergence for this series is $7-3$ or $4$.

Continue problem 3 on page 9.

 $E_2$

Work for problem 3(c)

$n^{th}$ term is $\dfrac{(-1)^n (x-5)^n}{2^n (n+2)}$

because $P(x)$ is an alternating series, the alternating series remainder theorem guarantees that error for n terms will be less than or equal to the |n+1 term|

for a $6^{th}$ degree polynomial, the error is $\leq |7^{th}$ term$|$

$$\text{error} \leq \left| \frac{(-1)^7 (x-5)^7}{2^7 (n+2)} \right| \quad x = 6$$

$$\text{error} \leq \left| \frac{(-1)^7 (6-5)^7}{2^7 (6+2)} \right|$$

$$\text{error} \leq \frac{1}{1024}$$

$$\therefore \frac{1}{1024} < \frac{1}{1000}$$

$\therefore$ the error for a $6^{th}$ degree polynomial is less than $\frac{1}{1000}$

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