AP® Calculus AB
2002 Sample Student Responses

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A_1

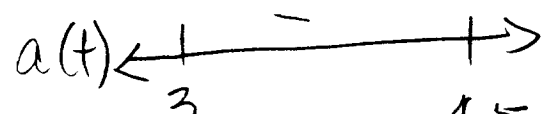
✓ Work for problem 3(a)

$a = \frac{dv}{dt} = \frac{\pi \cos\left(\frac{\pi t}{3}\right)}{3}$

$a(4) = \frac{\pi \cos\left(\frac{\pi 4}{3}\right)}{3}$

$a = \frac{-\pi}{6}$

✓ Work for problem 3(b)



$v(3) = \sin(\pi) = 0$

$v(4.5) = \sin\left(\frac{4.5\pi}{3}\right) = -1$

$a(t) < 0$

$v(t)$ is decreasing → statement I is correct

$a(t) < 0$ $v(t) \leq 0$ speed is $|v(t)|$ (no direction)

$|v(t)|$ is increasing since $v(t)$ and $a(t)$ are the same sign

↓

speed is increasing

↓

statement II is correct

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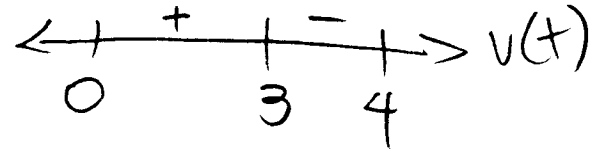
A_2

Work for problem 3(c)

$v(t) = 0$

$t = 3$

$t = 0$



$x(t) = \int v(t)\,dt$

$x_{tot}(t) = \int_0^3 v(t)\,dt + \int_4^3 v(t)\,dt$

$= \frac{6}{11} + \frac{3}{2\pi} = \frac{15}{2\pi}$

Work for problem 3(d)

$x(t) = \int v(t)\,dt$

$\frac{-3\cos\left(t\frac{\pi}{3}\right)}{\pi} + C = x(t)$

$\frac{-3}{\pi} + C = 2$

$C = \frac{3}{\pi} + 2$

$x(A) = \frac{-3\cos\left(\frac{4\pi}{3}\right)}{\pi} + \frac{3}{\pi} + 2$

$= \frac{9}{2\pi} + 2$

Work for problem 3(a)

$v(t) = \int a(t)\,dt$

$v'(t) = a(t)$

$a(t) = \frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right)$

$a(4) = \frac{\pi}{3}\cos\left(\frac{4\pi}{3}\right)$

$\approx -.524$

Work for problem 3(b)

The function $v(t) = \sin\left(\frac{\pi}{3}t\right)$ is negative from $3 < t < 4.5$, so velocity is decreasing. However, the function is concave up, so acceleration must be positive. When acceleration is positive, speed is increasing, so both statements are true.

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Work for problem 3(c)

$$TD = \int_0^4 |v(t)|\,dt$$
$$= \int_0^4 \left|\sin\left(\tfrac{\pi}{3}t\right)\right| dt$$
$$= 2.387 \text{ units}$$

Work for problem 3(d)

$$x(t) = \int v(t)\,dt$$
$$x(t) = \tfrac{3}{11}\int \sin\left(\tfrac{\pi}{3}t\right) dt + \tfrac{\pi}{3}$$
$$x(t) = \tfrac{3}{11}\int \sin u\,du$$
$$= -\tfrac{3}{11}\cos u + C$$
$$= -\tfrac{3}{11}\cos\left(\tfrac{\pi}{3}t\right) + C$$
$$2 = -\tfrac{3}{11}\cos\left(\tfrac{\pi}{3}\cdot 0\right) + C$$
$$2 = -\tfrac{3}{11}(1) + C$$
$$2.955 = C$$

$u = \frac{\pi}{3}t$

$du = \frac{\pi}{3}dt$

$$x(t) = -\tfrac{3}{11}\cos\left(\tfrac{\pi}{3}t\right) + 2.955$$
$$x(4) = -\tfrac{3}{11}\cos\left(\tfrac{4\pi}{3}\right) + 2.955$$
$$x(4) \approx 3.432$$