AP® Calculus AB
2002 Sample Student Responses

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A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ \int_{-5}^{1} (e^x - \ln x) \, dx = 1.223 \text{ units}^2 \]
Work for problem 1(b)

\[
\begin{align*}
R &= 4 - \ln x \\
Y &= 4 - e^x
\end{align*}
\]

\[
\int_{0.5}^{5} (4 - \ln(x))^2 - (4 - e^x)^2 \, dx = 23,609 \text{ units}^3
\]

Work for problem 1(c)

\[
\begin{align*}
\eta(x) &= f(x) - g(x) \\
&= e^x - \ln x \\
\eta'(x) &= e^x - \frac{1}{x}
\end{align*}
\]

For criticals:
\[
e^x - \frac{1}{x} = 0
\]

\[
\ln e^x = \ln x
\]

\[
x = \ln x
\]

\[
x = -\ln x
\]

\[
\ln x + x = 0
\]

\[
x = 0.567
\]

End pts: 0.5, 1

To determine the absolute minimum and maximum, I found any criticals (when \( \eta'(x) \) equals 0), and the end points. There was only one critical number, which occurred at \( x = 0.567 \). When I compared the values of each number (see chart), I found the minimum value to be 2.330 and the absolute maximum value to be 2.718 (or e).

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ A = \int_{\frac{1}{2}}^{u} e^x - \ln x \, dx = \begin{bmatrix} 1.223 \end{bmatrix} \, u^2 \]
\[(\text{using fnInt})\]
Work for problem 1(b)

\[ V = \pi \int_{\frac{1}{2}}^{1} (y - \ln x)^2 - (4 - e^x)^2 \, dx = \left[ \frac{23.610}{3} \right] \text{ cu. units} \]

(using data)

Work for problem 1(c)

\[ h'(x) = f'(x) - g'(x) \]
\[ h'(x) = e^x - \frac{1}{x} = 0 \]

\[ h(1) = e - \ln 1 = \text{MAX} \]
\[ e^x \text{ grows faster than } \ln x \text{ so on the interval } \frac{1}{2} \leq x \leq 1, \]

the greatest value of \( h(x) \) will be at \( x = 1 \).

\[ h\left(\frac{1}{2}\right) = \left[ e^{\frac{1}{2}} - \ln \frac{1}{2} \right] = \text{MIN} \]

since \( e^x \) grows faster than \( \ln x \) the min. value will

be at the very beginning of the interval at \( x = \frac{1}{2} \).