AP Calculus AB
1999 Sample Student Responses

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B1

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

Using $f'(0) = -3$ and the point $(0,2)$ on $f(x)$,

$y - 2 = -3(x - 0)$

$y - 2 = -3x$, $3x + y = 2$

Tangent line to f at $x = 0$:

$3x + y = 2$

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

We know that $f''(0) = 0$ which is one requirement of an inflection point because it is given to us. However, we cannot state this is an inflection point because we do not know whether $f''(x)$ changes signs at $x = 0$. (We do not know whether $f(x)$ changes concavity at $x = 0$.) Therefore, we do not have sufficient information.

Continue problem 4 on page 11.

B_2

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

$$g'(0) = e^{-2(0)}(3f(0) + 2f'(0))$$
$$= 3(2) + 2(-3)$$
$$= 6 - 6$$
$$= 0$$

Using $g'(0) = 0$ and the point $(0,4)$ on $g(x)$,

$$y - 4 = 0(x - 0)$$
$$y = 4$$

Tangent line to g at $x = 0$:

$$y = 4$$

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

$$g'(x) = e^{-2x}(3f(x) + 2f'(x))$$
$$g''(x) = (-2)e^{-2x}(3f(x) + 2f'(x)) +$$
$$(e^{-2x})(3f'(x) + 2f''(x))$$
$$= e^{-2x}[-6f(x) - 4f'(x) + 3f'(x) + 2f''(x)$$
$$= e^{-2x}[-6f(x) - f'(x) + 2f''(x)]$$

Checking for local maximum at $x=0$

$$g'(0) = 0$$
$$g''(0) = [-6f(0) - f'(0) + 2f''(0)]$$
$$= -12 + 3$$
$$= -9$$

Since g' is zero at $x=0$ and since g is concave downward at $x=0$ ($g''(0) = -9$), a local maximum exists at $x=0$.

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D1

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

$y = -3x + b$

$2 = -3(0) + b \rightarrow b = 2$

$y = -3x + 2$

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

No, we know that $f''(0) = 0$ but we do not know if there is a change in the concave direction of the graph of $f(x)$ which is required for an inflection point

Continue problem 4 on page 11.

D_2

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

$g'(0) = e^{-2(0)}(3f(0) + 2f'(0)) = e^{0}(3(2) + 2(-3))$

$= 1(6-6) = 1(0) = 0$

$y = 4$

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

$g''(x) = -2e^{-2x}(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$

$g''(x) = e^{-2x}(-6f(x) - 4f'(x)) + e^{2x}(3f'(x) + 2f''(x))$

$g''(x) = e^{-2x}(-6f(x) - 4f'(x) + 3f'(x) + 2f''(x))$

$g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$

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F_1

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

slope $= -3$

point $(0, 2)$

$y - 2 = -3(x - 0)$

$y = -3x + 2$

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

Yes, there is sufficient information to determine that f has a point of inflection at y = 0 because the second derivative is continuous, and it does have a zero at x = 0. Continuity implies that it is able to have a point of inflection at x = 0.

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(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

at the point (0,4)
find slope

$g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$g'(0) = 3f(0) + 2f'(0)$

$g'(0) = 3(2) + 2(-3)$

$g'(0) = 6 - 6 = 0$

$g'(0) = 0$

$y = 4$

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

$g'(x) = e^{-2x}(3f(x) + 2f'(x))$

$g''(x) = -2e^{-2x}(3f(x) + 2f'(x)) + e^{-2x}(3f'(x) + 2f''(x))$

$g''(x) = e^{-2x}[-6f(x) - 4f'(x) + 3f'(x) + 2f''(x)]$

$g''(x) = e^{-2x}[-6f(x) - f'(x) + 2f''(x)]$