AP Calculus AB
1999 Sample Student Responses

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A,

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves along the y-axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

$v(1.5) = 1.5 \sin(1.5^2) = 1.167$

The particle moves up when velocity is positive and down when velocity is negative. Since velocity is positive at $t = 1.5$, the particle is moving up

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(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

$a(t) = v'(t) = t \cdot 2t \cos(t^2) + \sin(t^2)$

$= 2t^2 \cos(t^2) + \sin(t^2)$

$a(1.5) = 2(1.5)^2 \cos(1.5^2) + \sin(1.5)^2 = -2.049$

The acceleration at $t = 1.5$ is -2.049 units/time^2

Since the acceleration at $t = 1.5$ is negative, this means that the velocity at $t = 1.5$ is not increasing but decreasing.

Continue problem 1 on page 5.

A_2

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

$$y(t) = \int v(t)\,dt = \int +\sin(t^2)\,dt$$

$$y(t) = -\tfrac{1}{2}\cos(t^2) + C$$

$$y(0) = 3 = -\tfrac{1}{2}\cos(0) + C$$

$$3 = -\tfrac{1}{2} + C$$

$$\tfrac{7}{2} = C$$

$$y(t) = -\tfrac{1}{2}\cos(t^2) + \tfrac{7}{2}$$

$$y(2) = -\tfrac{1}{2}\cos(4) + \tfrac{7}{2} = 3.827 \text{ units}$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

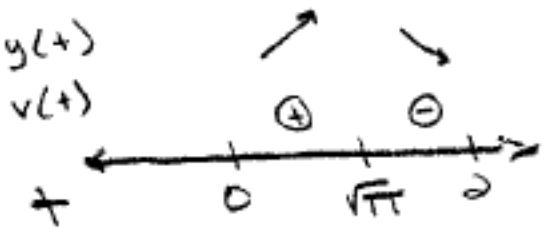
$$v(t) = +\sin(t^2) = 0$$

$t = 0$ $\quad \sin(t^2) = 0$

$$\sin^{-1}(0) = t^2$$

$$t^2 = 0, \pi \ldots$$

$$t = 0, \sqrt{\pi}$$



$$D = \int_0^{\sqrt{\pi}} (+\sin t^2)\,dt + \int_{\sqrt{\pi}}^{2} (+\sin t^2)\,dt$$

$$= \left(-\tfrac{1}{2}\cos(t^2)\right)\Big|_0^{\sqrt{\pi}} - \left(-\tfrac{1}{2}\cos(t^2)\right)\Big|_{\sqrt{\pi}}^{2}$$

$$= \left(-\tfrac{1}{2}\cos\pi + \tfrac{1}{2}\cos 0\right) - \left(-\tfrac{1}{2}\cos 4 + \tfrac{1}{2}\cos\pi\right)$$

total distance = 1.173 units

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D,

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves along the y-axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

$v(t) = t\sin(t^2)$

$v(1.5) = 1.5\sin(1.5^2)$

$v(1.5) = 1.17$

The particle is moving up at time $t = 1.5$ because its velocity at that time is positive

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(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

$v(t) = t\sin(t^2)$

$a(t) = \sin(t^2) + 2t^2\cos(t^2)$

$a(1.5) = -1.33$

The velocity of the particle is not increasing because the acceleration is negative.

Continue problem 1 on page 5.

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

$v(t) = t\sin(t^2)$

$y(t) = \int t\sin(t^2)\,dt$

$y(t) = -\frac{\cos(t^2)}{2} + C$

$y(3) = -\frac{\cos(0^2)}{2} + C$

$3 = -\frac{1}{2} + C$

$C = \frac{7}{2}$

$y(2) = -\frac{\cos(2^2)}{2} + \frac{7}{2}$

$y(2) = 3.83$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$\text{total distance} = \int_0^2 \left| t\sin(t^2)\,dt \right|$$

$$= 1.173$$

E,

CALCULUS AB

SECTION II

Time—1 hour and 30 minutes

Number of problems—6

Percent of total grade—50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves along the y-axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

$s(t) = \int t \sin(t^2)\,dt$ $\quad u = t^2$ $\quad -\frac{1}{2}\cos t^2 = 0$

$= -\cos$ $\quad du = 2t\,dt$ $\quad \cos t^2 = 0$

$2t \sin t^2\,dt$

$-\frac{1}{2}\cos t^2 + C$

The particle is moving up along the x-axis at time $t = 1.5$ because the position of the particle is positive

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(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

$v(t) = t \sin(t^2)$

$a(t) = t(2t \cos t^2) + \sin t^2$

$1.5(3 \cos 2.25) + \sin 2.25$

$+.778$

$a = -2.049$

The velocity is decreasing at $t = 1.5$ because the acceleration is negative at $t = 1.5$.

Continue problem 1 on page 5.

E_2

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

$$y(t) = -\tfrac{1}{2}\cos t^2 + C$$

$$3 = -\tfrac{1}{2}\cos 0^2 + C$$

$$3 = -\tfrac{1}{2} + C$$

$$3\tfrac{1}{2} = C$$

$$y(t) = -\tfrac{1}{2}\cos t^2 + 3\tfrac{1}{2}$$

$$-\tfrac{1}{2}\cos(4) + 3\tfrac{1}{2}$$

$$\boxed{= 3.827}$$

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

$$\int_0^2 -\tfrac{1}{2}\cos t^2 + 3\tfrac{1}{2}\,dt$$

$$\boxed{= 6.769 \text{ units}}$$

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