AP Calculus AB
1999 Sample Student Responses

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CALCULUS AB
SECTION II
Time — 1 hour and 30 minutes
Number of problems — 6
Percent of total grade — 50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

1. A particle moves along the y-axis with velocity given by \( v(t) = t \sin(t^2) \) for \( t \geq 0 \).
   (a) In which direction (up or down) is the particle moving at time \( t = 1.5 \)? Why?
   \[ v(1.5) = 1.5 \sin(1.5^2) = 1.167 \]
   The particle moves up when velocity is positive
   and down when velocity is negative. Since
   velocity is positive at \( t = 1.5 \), the particle
   is moving up.

(b) Find the acceleration of the particle at time \( t = 1.5 \). Is the velocity of the particle increasing at \( t = 1.5 \)?
   Why or why not?
   \[ a(t) = v'(t) = 1 \cdot 2t \cos(1.5^2) + 2 \cdot 1.5 \sin(1.5^2) \]
   \[ a(1.5) = 2(1.5)^2 \cos(1.5^2) + 2 \cdot 1.5 \sin(1.5^2) = -2.049 \]
   The acceleration at \( t = 1.5 \) is \( -2.049 \text{ units/s}^2 \).
   Since the acceleration at \( t = 1.5 \) is negative,
   this means that the velocity at \( t = 1.5 \) is not
   increasing but decreasing.

Continue problem 1 on page 5.
(c) Given that \( y(t) \) is the position of the particle at time \( t \) and that \( y(0) = 3 \), find \( y(2) \).

\[
y(t) = \int v(t) \, dt = \int +\sin(t^2) \, dt
\]

\[
y(t) = -\frac{1}{2} \cos(t^2) + C
\]

\[
y(0) = 3 = -\frac{1}{2} \cos(0) + C
\]

\[
\frac{3}{2} = C
\]

\[
y(t) = -\frac{1}{2} \cos(t^2) + \frac{3}{2}
\]

\[
y(2) = -\frac{1}{2} \cos(4) + \frac{3}{2} = 3.827 \text{ units}
\]

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(d) Find the total distance traveled by the particle from \( t = 0 \) to \( t = 2 \).

\[
v(t) = +\sin(t^2) = 0
\]

\[
y(t) = \int v(t) \, dt = \int 0 \, dt
\]

\[
y(t) = 0
\]

\[
0^2 = 0 \rightarrow 0
\]

\[
0 = 0 \rightarrow 0
\]

\[
0 = \int_0^2 (+\sin(t^2)) \, dt = \int_0^2 (+\sin(t^2)) \, dt
\]

\[
= \left[ -\frac{1}{2} \cos(t^2) \right]_0^2 - \left[ -\frac{1}{2} \cos(t^2) \right]_0^2
\]

\[
= (\frac{1}{2} \cos(4) + \frac{1}{2} \cos(0)) - (\frac{1}{2} \cos(4) + \frac{1}{2} \cos(0))
\]

\[
\text{total distance} = 1.173 \text{ units}
\]
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SECTION II
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1. A particle moves along the y-axis with velocity given by \( v(t) = t \sin(t^2) \) for \( t \geq 0 \).

   (a) In which direction (up or down) is the particle moving at time \( t = 1.5 \) ? Why?

   \[
   v(t) = t \sin(t^2)
   \]

   \[
   v(1.5) = 1.5 \sin(1.5^2)
   \]

   \[
   v(1.5) = 1.14
   \]

   The particle is moving up at time \( t = 1.5 \) because its velocity at that time is positive.

   (b) Find the acceleration of the particle at time \( t = 1.5 \). Is the velocity of the particle increasing at \( t = 1.5 \) ? Why or why not?

   \[
   v(t) = t \sin(t^2)
   \]

   \[
   a(t) = \sin(t^2) + 2t^2 \cos(t^2)
   \]

   \[
   a(1.5) = -1.33
   \]

   The velocity of the particle is not increasing because the acceleration is negative.

Continue problem 1 on page 5.
(c) Given that \( y(t) \) is the position of the particle at time \( t \) and that \( y(0) = 3 \), find \( y(2) \).

\[
y(t) = \int tsin(t^2) \, dt
\]

\[
y(t) = -\frac{\cos(t^2)}{2} + C
\]

\[
y(2) = \frac{-\cos(2^2)}{2} + \frac{7}{2}
\]

\[
y(2) = 3.83
\]

(d) Find the total distance traveled by the particle from \( t = 0 \) to \( t = 2 \).

\[
\text{total distance} = \left| \int_0^2 t \sin(t^2) \, dt \right|
\]

\[
= 1.173
\]
1. A particle moves along the y-axis with velocity given by \( v(t) = t \sin(t^2) \) for \( t \geq 0 \).
   (a) In which direction (up or down) is the particle moving at time \( t = 1.5 \)? Why?

   \[
   s(t) = \int t \sin(t^2) \, dt
   \]

   \[
   = \left[ \frac{-\cos(t^2)}{2t} \right] + c
   \]

   \[
   = -\cos + c
   \]

   The particle is moving up along the y-axis at time \( t = 1.5 \) because the position of the particle is positive.

(b) Find the acceleration of the particle at time \( t = 1.5 \). Is the velocity of the particle increasing at \( t = 1.5 \)? Why or why not?

   \[
   v(t) = t \sin(t^2)
   \]

   \[
   a(t) = \left( \frac{d}{dt} \sin(t^2) \right) + \sin(t^2)
   \]

   \[
   = 1.5 \left( 3 \cos(2.25) \right) + \sin(2.25)
   \]

   \[
   = 1.5 \left( 3 \times 0.7071 \right) + 0.7071
   \]

   \[
   = 7.778
   \]

   \[
   a = -2.049
   \]

   The velocity is decreasing at \( t = 1.5 \) because the acceleration is negative at \( t = 1.5 \).
(c) Given that \( y(t) \) is the position of the particle at time \( t \) and that \( y(0) = 3 \), find \( y(2) \).

\[
y(t) = -\frac{1}{2} \cos t^2 + c
\]

\[3 = -\frac{1}{2} \cos 0^2 + c\]

\[3 = -\frac{1}{2} + c\]

\[3 + \frac{1}{2} = c\]

\[y(t) = -\frac{1}{2} \cos t^2 + 3\frac{1}{2} - \frac{1}{2} \cos (4t) + 3\frac{1}{2}\]

\[= 3.827\]

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(d) Find the total distance traveled by the particle from \( t = 0 \) to \( t = 2 \).

\[
\int_{0}^{2} -\frac{1}{2} \cos t^2 + 3\frac{1}{2} \ dt
\]

\[= 6.769 \text{ units}\]