



AP[®] Calculus AB 2001 Sample Student Responses

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$y = f(x)$$

$$f(3) = \frac{1}{4}$$

$$\frac{dy}{dx} = y^2(6-2x)$$

$$\frac{d^2y}{dx^2} = (y^2)(-2) + (6-2x) \cdot (2y \cdot y')$$

$$\frac{d^2y}{dx^2} = -2y^2 + 12y \cdot y' - 4xy \cdot y'$$

$$\frac{d^2y}{dx^2} \left(3, \frac{1}{4}\right) = -2\left(\frac{1}{4}\right)^2 + 12\left(\frac{1}{4}\right) \cdot y' - 4(3)\left(\frac{1}{4}\right) \cdot y'$$

$$= -\frac{1}{8} + 3y' - 3y'$$

$$\frac{d^2y}{dx^2} \left(3, \frac{1}{4}\right) = -\frac{1}{8}$$

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NO CALCULATOR ALLOWED

Work for problem 6(b)

$$\frac{dy}{dx} = y^2(6-2x)$$

$$\frac{dy}{y^2} = (6-2x) dx$$

$$\frac{1}{y^2} dy = (6-2x) dx$$

$$\int y^{-2} dy = \int (6-2x) dx$$

$$-y^{-1} = 6x - x^2 + C$$

$$-\frac{1}{y} = 6x - x^2 + C$$

$$\frac{1}{y} = x^2 - 6x - C$$

$$y = \frac{1}{x^2 - 6x + C}$$

$$\frac{1}{4} = \frac{1}{(3)^2 - 6(3) + C}$$

$$\frac{1}{4} = \frac{1}{C-9}$$

$$C-9=4$$

$$C=13$$

$$y = \frac{1}{x^2 - 6x + 13}$$

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NO CALCULATOR ALLOWED

Work for problem 6(a)

$$y' = y^2(6-2x)$$

$$y' = 6y^2 - 2y^2x$$

$$y'' = 12yy' - 2y^2 - 4yy'x \leftarrow \frac{d^2y}{dx^2}$$

$$y'' = 12y(y^2(6-2x)) - 2y^2 - 4y(y^2(6-2x)) \text{ at } (3, \frac{1}{4})$$

$$y'' = 12(\frac{1}{4})(\frac{1}{8}(6-6)) - 2(\frac{1}{8}) - 4(\frac{1}{4})(\frac{1}{8}(6-6))$$

$$y'' = 3(0) - \frac{2}{8} - 0$$

$$y'' = -\frac{2}{8} = -\frac{1}{4}$$

$$\frac{d^2y}{dx^2} \text{ at point } (3, \frac{1}{4}) = \boxed{-\frac{1}{4}}$$

$$(2y^2)(x) \quad UV' + VU'$$

$$2y^2 + 4yy'x$$

$$f(3) = \frac{1}{4}$$

Work for problem 6(b)

$$\frac{dy}{dx} = y^2(6-2x)$$

$$\int \frac{dy}{y^2} = \int 6-2x dx$$

$$\int y^{-2} dy = \int 6-2x dx$$

$$-y^{-1} = 6x - \frac{2x^2}{2} + C$$

$$-\frac{1}{y} = 6x - x^2 + C$$

$$-\frac{1}{\frac{1}{4}} = 6(3) - 9 + C$$

$$-4 = 18 - 9 + C$$

$$-4 = -9 + C$$

$$-4 + 9 = C$$

$$5 = C$$

$$-\frac{1}{y} = \frac{6x - x^2 + 5}{1}$$

$$-1 = y(6x - x^2 + 5)$$

$$\frac{-1}{6x - x^2 + 5} = y$$