



AP[®] Calculus AB 2001 Sample Student Responses

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NO CALCULATOR ALLOWED

CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

A₁

No calculator is allowed for these problems.

Work for problem 4(a)

$$h'(x) = 0 = \frac{x^2 - 2}{x}$$

$$0 = x^2 - 2$$

$$x = \pm\sqrt{2}$$

\therefore h has horizontal tangents at $x = \sqrt{2}$ and $x = -\sqrt{2}$

$$h'(x) \begin{array}{c} - \quad 0 \quad + \quad - \quad 0 \quad + \\ \leftarrow \quad \quad \quad \rightarrow \\ -\sqrt{2} \quad 0 \quad \sqrt{2} \end{array}$$

\therefore h has a local minimum at $x = \sqrt{2}$ and $x = -\sqrt{2}$

Work for problem 4(b)

$$h''(x) = \frac{2x \cdot x - (x^2 - 2)}{x^2}$$

$$= \frac{2x^2 - x^2 + 2}{x^2}$$

$$= \frac{x^2 + 2}{x^2}$$

$$h''(x) \begin{array}{c} + \quad 0 \quad + \\ \leftarrow \quad \quad \rightarrow \\ 0 \end{array}$$

\therefore h is concave up on the intervals $(-\infty, 0)$ and $(0, \infty)$

4 4 4 4 4 4 4 4 4 4

NO CALCULATOR ALLOWED

A₂

Work for problem 4(c)

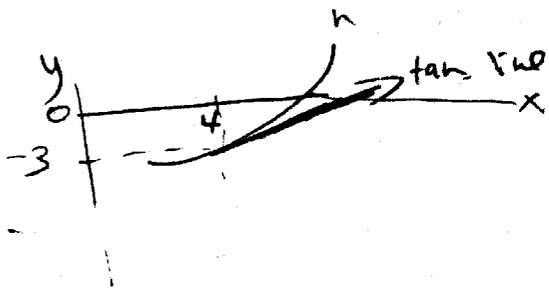
$$h'(4) = \frac{4^2 - 2}{4} = \frac{14}{4} = \frac{7}{2}$$

$$h(4) = -3$$

$$y + 3 = \frac{7}{2}(x - 4)$$

Work for problem 4(d)

The tangent line to the graph of h at $x = 4$ lies below the graph of h for $x > 4$ because h is concave up on the interval $(0, \infty)$.



NO CALCULATOR ALLOWED

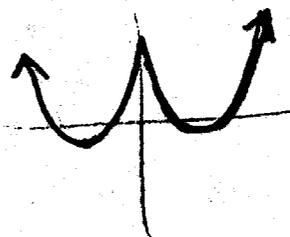
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$h'(x) = 0$$

$$x \neq 0$$

$$\frac{x^2 - 2}{x} = 0$$

$$x^2 - 2 = 0$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$h''(x) = \frac{2x^2 - x^2 + 2}{x^2}$$

$$h''(+\sqrt{2}) = \frac{4 - 2 + 2}{2}$$

$$h''(-\sqrt{2}) = \frac{4}{2} = 2$$

$$h''(\sqrt{2}) +$$

$$h''(-\sqrt{2}) +$$

at $+\sqrt{2}$, minimum

+ at $-\sqrt{2}$, minimum

because the second derivative is +, which means the slope is increasing

Work for problem 4(b)

$$x \neq 0 \quad \frac{2x^2 - x^2 + 2}{x^2} > 0$$

~~$$2x^2 - x^2 + 2 > 0$$~~

$$x^2 + 2 > 0$$

$$x^2 > -2$$

$$x^2$$

(h) is concave up for

all values, as

x^2 must always be greater than -2

NO CALCULATOR ALLOWED

D₂

Work for problem 4(c)

$$h(4) = -3$$

$$h'(x) = \frac{x^2 - 2}{x}$$

$$h'(4) = \frac{16 - 2}{4} = \frac{14}{4} = \frac{7}{2}$$

$$y + 3 = \frac{7}{2}(x - 4)$$

$$y = \frac{7}{2}x - 14 - 3$$

$$y = \frac{7}{2}x - 17$$

Work for problem 4(d)

below, because the
graph is concave

up

