AP Calculus AB
2000 Student Samples

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B.

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

WATER LEAKS OUT AT A RATE OF:

$\frac{dV}{dt}$ rate for leakage

$$\frac{dV}{dt} = -\sqrt{t+1} \text{ gallons/min.}$$

$$\int_0^3 dV = \int_0^3 -\sqrt{t+1}\, dt$$

$$V\Big]_0^3 = -\left[\frac{2(t+1)^{\frac{3}{2}}}{3}\right]_0^3 = -\left(\frac{2(8)}{3} - \frac{2}{3}\right)$$

$$= -\frac{14}{3} \text{ gallons}$$

$\frac{14}{3}$ gallons leak out

Work for problem 4(b)

rate at which volume in tank is changing $\rightarrow \frac{dV}{dt} = 8 - \sqrt{t+1}$ gallons/min.

INITIAL

$$\int_0^3 dV = \int_0^3 (8 - \sqrt{t+1})\, dt$$

$$V\Big]_0^3 = \left[8t - \frac{2(t+1)^{\frac{3}{2}}}{3}\right]_0^3$$

$$= \left(24 - \frac{2(8)}{3}\right) - \left(-\frac{2}{3}\right)$$

$$= \frac{56}{3} + \frac{2}{3} = \frac{58}{3} \text{ gallons}$$

$\frac{58}{3} + 30$

$= \frac{148}{3}$ gallons

ANSWER

Continue problem 4 on page 11.

B_2

Work for problem 4(c)

$$\frac{d}{dt}A(t) = 8 - \sqrt{t+1}$$

$$\int dA(t) = \int \left(8 - \sqrt{t+1}\right) dt$$

$$A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + C$$

$$30 = 8(0) - \frac{2(0+1)^{\frac{3}{2}}}{3} + C$$

$$C = 30 + \frac{2}{3} = \frac{92}{3}$$

$$\boxed{A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + \frac{92}{3}}$$

Work for problem 4(d)

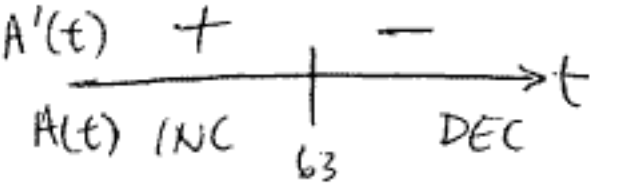
$$8 - \sqrt{t+1} = 0$$

$$-\sqrt{t+1} = -8$$

$$\sqrt{t+1} = 8$$

$$t+1 = 64$$

$$t = 63$$



AT $\boxed{t = 63 \text{ minutes}}$ amount of water is maximum because of the FIRST DERIVATIVE TEST.

4 4 4 4 4 4 4 4 4 4

D1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

Pumped – 8 gpm
leaks – $\sqrt{t+1}$ gpm
t = 0, 30 gallons

$$\int_0^3 \sqrt{t+1}\,dt = \int_0^3 (t+1)^{\frac{1}{2}}\,dt$$

$u = t+1$
$du = dt$

$$= \frac{2}{3}(t+1)^{\frac{3}{2}}\Big]_0^3$$

$$= \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}}$$

$$= \frac{2}{3}(8) - \frac{2}{3}$$

$$= \frac{16}{3} - \frac{2}{3}$$

$$= \frac{14}{3} \text{ gallons}$$

Work for problem 4(b)

$$\frac{8 \text{ gallons}}{1 \text{ min}}\left(3 \text{ minutes}\right) = 24 \text{ gallons} - \frac{14}{3} \text{ gallons} =$$

$$\frac{72}{3} - \frac{14}{3} = \boxed{\frac{58}{3} \text{ gallons}}$$

24
3
72

6 7 2
14
58

Continue problem 4 on page 11.

4 4 4 4 4 4 4 4 4 4

D_2

Work for problem 4(c)

$$A(t) = 8t - \int_0^t (t+1)^{\frac{1}{2}} dt$$

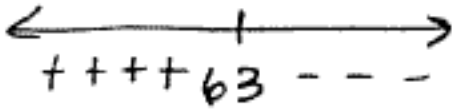
Work for problem 4(d)

$$A'(t) = 8 - (t+1)^{\frac{1}{2}} = 0$$

$$\left(\sqrt{t+1}\right)^2 = (8)^2$$

$$t+1 = 64$$

$$\boxed{t = 63}$$



When t is 63, the graph A(t) reaches a maximum (goes from positive to negative). So, the amount of water is at its maximum in the tank when $t = 63$.

GO ON TO THE NEXT PAGE.

4 4 4 4 4 4 4 4 4 4

F1

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

$$\int_0^3 \sqrt{t+1}\,dt$$

$$\int_0^3 (t+1)^{1/2}\,dt$$

$$\left[\frac{2}{3}(t+1)^{3/2}\right]_0^3$$

4

$$\left(\frac{2}{3}\right)(4)^{3/2} - \left(\frac{2}{3}\right)(1)^{3/2}$$

$$\left(\frac{2}{3}\right)(8) - \frac{2}{3} = \frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3} \text{ gallons}}$$

Work for problem 4(b)

$$54 - \frac{14}{3} = \frac{162}{3} - \frac{14}{3} = \frac{148}{3} = \boxed{49\frac{1}{3} \text{ gallons}}$$

Continue problem 4 on page 11.

4 4 4 4 4 4 4 4 4 4

F_2

Work for problem 4(c)

$$A(t) = \left[(30 + 8t)\right] - \left[\int_0^3 \sqrt{t+1}\, dt\right]$$

Work for problem 4(d)

$(t+1)^{1/2}$

$\frac{1}{2}(t+1)^{-1/2} = 0$