



AP Calculus AB 2000 Student Samples

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CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

WATER LEAKS OUT AT A RATE OF:

~~rate of water~~
~~leakage~~
 $\frac{dv}{dt}$ rate for leakage

$$\frac{dv}{dt} = -\sqrt{t+1} \text{ gallons/min.}$$

$$\int_0^3 dv = \int_0^3 -\sqrt{t+1} dt$$

$$V \Big|_0^3 = - \left[\frac{2(t+1)^{\frac{3}{2}}}{3} \right]_0^3 = - \left(\frac{2(8)}{3} - \frac{2}{3} \right)$$

$$= -\frac{14}{3} \text{ gallons}$$

$\frac{14}{3}$ gallons leak out

Work for problem 4(b)

rate at which volume in tank is changing $\rightarrow \frac{dv}{dt} = 8 - \sqrt{t+1}$ gallons/min.

$$\int_0^3 dv = \int_0^3 (8 - \sqrt{t+1}) dt$$

$$V \Big|_0^3 = \left[8t - \frac{2(t+1)^{\frac{3}{2}}}{3} \right]_0^3$$

$$= \left(24 - \frac{2(8)}{3} \right) - \left(-\frac{2}{3} \right)$$

$$= \frac{56}{3} + \frac{2}{3} = \frac{58}{3} \text{ gallons}$$

INITIAL

$$\frac{58}{3} + 30$$

$$= \frac{148}{3} \text{ gallons}$$

ANSWER

$$\frac{72}{56}$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$\frac{d}{dt} A(t) = 8 - \sqrt{t+1}$$

$$\int dA(t) = \int (8 - \sqrt{t+1}) dt$$

$$A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + \frac{92}{3}$$

$$A(t) = 8t - \frac{2(t+1)^{\frac{3}{2}}}{3} + C$$

$$30 = 8(0) - \frac{2(0+1)^{\frac{3}{2}}}{3} + C$$

$$C = 30 + \frac{2}{3} = \frac{92}{3}$$

Work for problem 4(d)

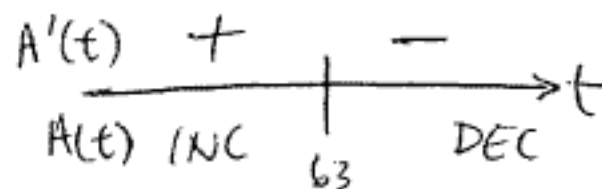
$$8 - \sqrt{t+1} = 0$$

$$-\sqrt{t+1} = -8$$

$$\sqrt{t+1} = 8$$

$$t+1 = 64$$

$$t = 63$$



AT $t = 63$ minutes amount of water is maximum because of the FIRST DERIVATIVE TEST.

CALCULUS AB

D1

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

pumped - 8 gpm
 leaks - $\sqrt{t+1}$ gpm
 $t=0$, 30 gallons

$$\int_0^3 \sqrt{t+1} dt = \int_0^3 (t+1)^{\frac{1}{2}} dt \quad u=t+1 \quad du=dt$$

$$= \left. \frac{2}{3}(t+1)^{\frac{3}{2}} \right|_0^3$$

$$= \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(1)^{\frac{3}{2}}$$

$$= \frac{2}{3}(8) - \frac{2}{3}$$

$$= \frac{16}{3} - \frac{2}{3}$$

$$= \frac{14}{3} \text{ gallons}$$

Work for problem 4(b)

$$\frac{8 \text{ gallons}}{1 \text{ min}} \left(\frac{3 \text{ minutes}}{1} \right) = 24 \text{ gallons} - \frac{14}{3} \text{ gallons} =$$

$$\frac{24}{3} = 8$$

$$\frac{72}{3} - \frac{14}{3} = \boxed{\frac{58}{3} \text{ gallons}}$$

$$\frac{67.2}{14} = 4.8$$

Continue problem 4 on page 11.

Work for problem 4(c)

$$A(t) = 8t - \int_0^t (t+1)^{\frac{1}{2}} dt$$

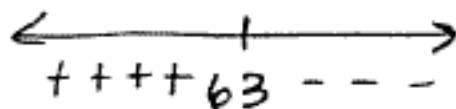
Work for problem 4(d)

$$A'(t) = 8 - (t+1)^{\frac{1}{2}} = 0$$

$$(\sqrt{t+1})^2 = (8)^2$$

$$t+1 = 64$$

$$t = 63$$



When t is 63, the graph $A(t)$ reaches a maximum (goes from positive to negative). So, the amount of water is at its maximum in the tank when $t = 63$.

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

F₁

No calculator is allowed for these problems.

Work for problem 4(a)

$$\int_0^3 \sqrt{t+1} dt$$

$$\int_0^3 (t+1)^{1/2} dt$$

$$\left[\frac{2}{3}(t+1)^{3/2} \right]_0^3$$

$$\left(\frac{2}{3} \right) (4)^{3/2} - \left(\frac{2}{3} \right) (1)^{3/2}$$

$$\left(\frac{2}{3} \right) (8) - \frac{2}{3} = \frac{16}{3} - \frac{2}{3} = \boxed{\frac{14}{3} \text{ gallons}}$$

Work for problem 4(b)

$$54 - \frac{14}{3} = \frac{162}{3} - \frac{14}{3} = \frac{148}{3} = \boxed{49 \frac{1}{3} \text{ gallons}}$$

Continue problem 4 on page 11.

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Work for problem 4(c)

$$A(t) = \left[(30 + 8t) - \int_0^3 \sqrt{t+1} dt \right]$$

Work for problem 4(d)

$$(t+1)^{1/2}$$

$$\frac{1}{2}(t+1)^{-1/2} = 0$$

GO ON TO THE NEXT PAGE.