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CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

WATER LEAKS OUT AT A RATE OF:

\[
\frac{dv}{dt} = -\sqrt{t+1} \text{ gallons/min.}
\]

\[
\int_{0}^{3} dv = \int_{0}^{3} -\sqrt{t+1} \, dt
\]

\[
V_{3}^{0} = \left[ -\frac{2(t+1)^{3/2}}{3} \right]_{0}^{3} = \left( -\frac{2(8)}{3} - \left( -\frac{2}{3} \right) \right)
\]

\[
= -\frac{14}{3} \text{ gallons}
\]

\[
\frac{14}{3} \text{ gallons leak out}
\]

Work for problem 4(b)

rate at which volume in tank is changing:

\[
\frac{dv}{dt} = 8 - \sqrt{t+1} \text{ gallons/min.}
\]

\[
\int_{0}^{3} dv = \int_{0}^{3} (8 - \sqrt{t+1}) \, dt
\]

\[
V_{3}^{0} = \left[ 8t - \frac{2(t+1)^{3/2}}{3} \right]_{0}^{3}
\]

\[
= (24 - \frac{2(8)}{3}) - \left( -\frac{2}{3} \right)
\]

\[
= \frac{56}{3} + \frac{2}{3} = \frac{58}{3} \text{ gallons}
\]

initial: \[
\frac{58}{3} + 30 = \frac{148}{3} \text{ gallons}
\]

answer: \[
\frac{58}{3} \text{ gallons}
\]

continue problem 4 on page 11.
Work for problem 4(c)

\[
\frac{d}{dt} A(t) = 8 - \sqrt{t+1} \\
\int dA(t) = \int (8 - \sqrt{t+1}) \, dt \\
A(t) = 8t - \frac{2(t+1)^{3/2}}{3} + C \\
30 = 8(0) - \frac{2(0+1)^{3/2}}{3} + C \\
C = 30 + \frac{2}{3} = \frac{92}{3}
\]

Work for problem 4(d)

\[
8 - \sqrt{t+1} = 0 \\
-\sqrt{t+1} = -8 \\
\sqrt{t+1} = 8 \\
t + 1 = 64 \\
t = 63
\]

At \( t = 63 \) minutes, the amount of water is maximum because of the first derivative test.
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

**Work for problem 4(a)**

Pumped = 8 gpm
Leaks = \( \sqrt{t+1} \) gpm
\( t = 0, 30 \text{ gallons} \)

\[
\int_0^3 \sqrt{t+1} \, dt = \int_0^3 (t+1)^{1/2} \, dt
\]
\[
\frac{2}{3} (t+1)^{3/2} \bigg|_0^3
\]
\[
= \frac{2}{3} (4)^{3/2} - \frac{2}{3} (1)^{3/2}
\]
\[
= \frac{2}{3} (8) - \frac{2}{3}
\]
\[
= \frac{16}{3} - \frac{2}{3}
\]
\[
= \frac{14}{3} \text{ gallons}
\]

**Work for problem 4(b)**

\[
\frac{8 \text{ gallons}}{1 \text{ min}} \left( \frac{3 \text{ minutes}}{} \right) = 24 \text{ gallons} - \frac{14}{3} \text{ gallons} = \frac{72}{3} - \frac{14}{3} = \frac{58}{3} \text{ gallons}
\]

Continue problem 4 on page 11.
Work for problem 4(c)

\[ A(t) = 8t - \int_0^t (t+1)^{\frac{1}{2}} \, dt \]

Work for problem 4(d)

\[ A'(t) = 8 - (t+1)^{\frac{1}{2}} = 0 \]

\[ \left(\sqrt{t+1}\right)^2 = 8^2 \]

\[ t + 1 = 64 \]

\[ t = 63 \]

When \( t \) is 63, the graph \( A(t) \) reaches a maximum (goes from positive to negative). So, the amount of water is at its maximum in the tank when \( t = 63 \).
CALCULUS AB
SECTION II, Part B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

Work for problem 4(a)

\[
\int_0^3 \sqrt{t^2 + 1} \, dt
\]

\[
\frac{3}{2} (t + 1)^{3/2} \bigg|_0^3
\]

\[
= \frac{3}{2} (4^{3/2} - 1^{3/2}) = \frac{3}{2} (8 - 1) = \frac{14}{3}
\]

Work for problem 4(b)

\[
54 - \frac{14}{3} = \frac{162}{3} - \frac{14}{3} = \frac{148}{3} = 49 \frac{1}{3} \text{ gallons}
\]

Continue problem 4 on page 11.
Work for problem 4(c)

\[ A(t) = \left[ (30 + 8t) \right] - \left[ \frac{3}{2} \int t + 1 \, dt \right] \]

Work for problem 4(d)

\[ \left( t + 1 \right)^{\frac{1}{2}} \]

\[ \frac{1}{2} (t + 1)^{-\frac{1}{2}} = 0 \]