AP Calculus AB
2000 Student Samples

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A,

Work for problem 3(a)

a. minimum - (-1,)
Candidate theorem says f may have a minimum on an open interval where $f'(x)=0$ or does not exist. $f'(x)$ exists everywhere and equals 0 at -5, -1, and 5. $f'(x)$ changes sign at -1 from negative to positive, which means f(x) has stopped decreasing and started increasing. This is the only candidate to change sign in this manner.

Work for problem 3(b)

b. Maximum - (-5)
The Candidate theorem says that f may have a maximum on an open interval where $f'(x)=0$ or does not exist. $f'(x)$ exists everywhere and equals 0 at -5, -1, and 5. $f'(x)$ changes sign from negative to positive at -1, does not change sign at 5, and changes sign from positive to negative at -5. Sign change from positive to negative indicates a relative maximum, because f(x) has stopped increasing and started decreasing.

Continue problem 3 on page 9.

A2

Work for problem 3(c)

c. $f''(x) < 0$ - $-7 < x < -3$, $2 < x < 3$, $3 < x < 5$

Work for problem 3(d)

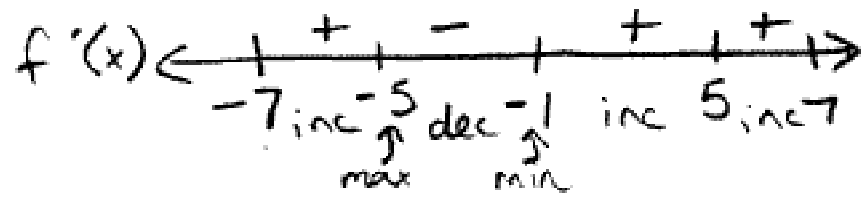
d. max - (7)

The Candidate Theorem tell's us that when f may have its absolute maximum when $f'(x) = 0$ and at the endpoints of a closed interval. This leaves us (-7, -5, -1, 5, 7). f(x) must be increasing up to the max (eliminates -7 and -1) and cannot increase more immediately after the max (eliminates 5). We know f(x) is increasing when f'(x) is posi To decide between -5 and 7, we can see that there is more increase of f(x) (positive area under the curve of f'(x)) than decrease (negative area) between -5 and 7 So f(x) must have increased more than decreased between -5 and 7, making f(7) larger than f(-5)

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)



f attains relative minima at x = −7, because it is an end point from which f increases, and at x = −1, because f' changes from negative to positive here, indicating a change in f from decreasing to increasing, indicative of a local minimum

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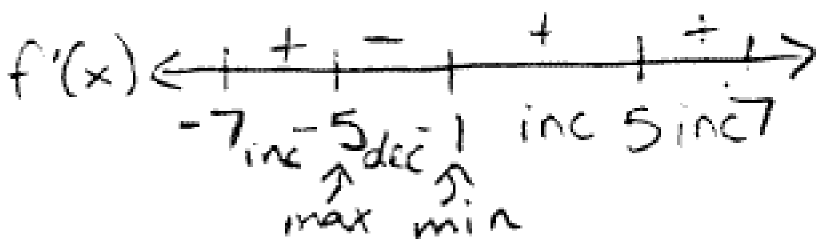
Work for problem 3(b)



f attains relative maxima at x = 7, because it is an end point of the function to which f increases, and at x = −5, because f' changes from positive to negative here, indicating a change in f from increasing to decreasing, indicative of a local maximum

Continue problem 3 on page 9.

Work for problem 3(c)

$f'(x)$ decreases $(-7,-3) \cup (2,5)$
$f'(x)$ increases $(-3,2) \cup (5,7)$

$f''(x) < 0$ where $f'(x)$ decreases

$\therefore f''(x) < 0$ on the intervals $(-7,-3) \cup (2,5)$

Work for problem 3(d)

$x = 7$

Absolute maxima occur where $f'(x) = 0$ and is changing from positive to negative OR at the end points of the function. The area under the given curve $f'(x)$, which is $f(x)$, increases the most $(-1, 7)$, more than $(-7,-5)$ where the other local maximum occurs at $x = -5$. Thus the absolute maximum occurs at $x = 7$.

END OF PART A OF SECTION II

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Work for problem 3(a)

at $x=-1$, f attains a relative minimum, because $f'(-1)=0$ and $f'(x) \rightarrow \underset{-1}{\overset{-\quad+}{\longleftrightarrow}}$
therefore at $x=-1$ there is a relative minimum

Work for problem 3(b)

at $x=-5$, f attains a relative maximum because $f'(-5)=0$ and
$f'(x) \rightarrow \underset{-5}{\overset{+\quad-}{\longleftrightarrow}}$, therefore at $x=-5$ there is a relative maximum

Continue problem 3 on page 9.

Work for problem 3(c)

$\left.\begin{array}{l}[-7,-3] \\ [2,5]\end{array}\right\} f''(x) < 0$

The graph of f(x) is concave down during these intervals

Work for problem 3(d)

at x=7 the graph of f attains its absolute maximum because ~~the~~ f'(x) has been positive for most of the interval, therefore at x=7, f attains its highest point.

END OF PART A OF SECTION II

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