AP® Calculus BC
2002 Sample Student Responses
Form B

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A1

NO CALCULATOR ALLOWED

Work for problem 6(a)

$\ln\left(\frac{1}{1-x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{x^n}{n}$

$\ln\left(\frac{1}{1+3x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{(-3x)^n}{n}$

$\left|\frac{(-3x)^{n+1}}{n+1} \cdot \frac{n}{(-3x)^n}\right| = \left|\frac{(-3x)^n}{n+1}\right| =$

$|x| < \frac{1}{3}$

$-\frac{1}{3} < x \leq \frac{1}{3}$

(when $x = \frac{1}{3}$ Converge
when $x = -\frac{1}{3}$ diverge

Work for problem 6(b)

$\ln\left(\frac{1}{1-x}\right) \Rightarrow \sum_{n=1}^{\infty} \frac{x^n}{n}$

when $x = -1$

$\ln\left(\frac{1}{2}\right) = -\ln 2$

A2

NO CALCULATOR ALLOWED

Work for problem 6(c)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converge at $0 < p \leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ diverge at $0 < p \leq \frac{1}{2}$

$\therefore\ 0 < p \leq \frac{1}{2}$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ converge by alternating $p = \frac{1}{2}$ series test

$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges by p-series test

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Work for problem 6(d)

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges $\quad \propto p \leq 1$

$\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ converges $\quad \frac{1}{2} < p$

$\therefore\ \frac{1}{2} < p \leq 1$

$p = 1$

$\sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test.

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series test

D1

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NO CALCULATOR ALLOWED

Work for problem 6(a)

Since $\ln\left(\frac{1}{1-x}\right) = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots \sum_{n=1}^{\infty} \frac{x^n}{n}$,

$$\ln\left(\frac{1}{1+3x}\right) = \ln\left(\frac{1}{1-(-3x)}\right) = -3x + \frac{(-3x)^2}{2} + \frac{(-3x)^3}{3} \cdots \sum_{n=1}^{\infty} \frac{(-3)^n x^n}{n}$$

Interval of convergence:

$|-3x| < 1$

$x > -\frac{1}{3}$

$x < \frac{1}{3}$

$$-\frac{1}{3} < x \leq \frac{1}{3}$$

Work for problem 6(b)

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \Rightarrow$ alternating harmonic series $\Rightarrow$ converges

$= -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} \cdots =$ **0**

NO CALCULATOR ALLOWED

Work for problem 6(c)

$p = \frac{1}{3}$,

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ will converge b/c of its similarity to the alternating harmonic series.

$\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$ is a divergent p-series since $p < 1$.

Work for problem 6(d)

$p = 1$.

$\sum_{n=1}^{\infty} \frac{1}{n^1} \Rightarrow$ divergent harmonic series.

$\sum_{n=1}^{\infty} \frac{1}{n^2} \Rightarrow$ convergent p-series since $p > 1$.