AP® Calculus AB
2002 Sample Student Responses
Form B

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A1

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NO CALCULATOR ALLOWED

Work for problem 5(a)

1) Solving the equation

$\frac{dy}{dx} = \frac{3-x}{y}$

$y\,dy = (3-x)\,dx$

$\int y\,dy = \int (3-x)\,dx$

$\frac{y^2}{2} = 3x - \frac{x^2}{2} + C \qquad y^2 = 6x - x^2 + C$. General solution.

2) Because $y=-2$ is tangent to $f(x)$ at $(x_0, -2)$,

$\left.\frac{dy}{dx}\right|_{\substack{x=x_0 \\ y=-2}} = 0. \qquad \frac{3-x_0}{-2} = 0$

$x_0 = 3.$

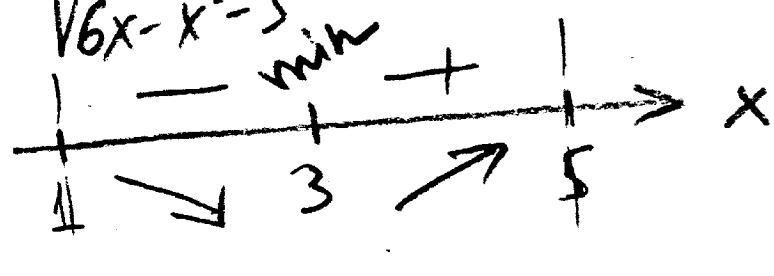
3) Particular solution $y = -\sqrt{6x \cdot}$

$(3,-2) \quad 4 = 6\cdot 3 - 9 + C$

$C = \bullet - 5.$

4) $y' = \frac{-(3-x)}{\sqrt{6x-x^2-5}} = \frac{x-3}{\sqrt{6x-x^2-5}}$

at $x=3$



at $x=3$ $y=f(x)$ has a local minimum.

A2

NO CALCULATOR ALLOWED

Work for problem 5(b)

$$\frac{dy}{dx} = \frac{3-x}{y}$$

$y^2 = 6x - x^2 + C$ as found in a)1).

$y(6) = -4$ $\quad 16 = 6\cdot(+6) - 36 + C$

$\therefore C = 16$

$y^2 = 6x - x^2 + 16$

$y = -\sqrt{6x - x^2 + 16}$

GO ON TO THE NEXT PAGE.

D1

5 5 5 5 5 5 5 5 5 5

NO CALCULATOR ALLOWED

Work for problem 5(a)

14 x 15

$y' = \frac{3-x}{y}$

$\frac{dy}{dx} = \frac{3-x}{y}$

$0 = \frac{3-x}{y}$

$0 = 3 - x$

~~Point of tangency~~ $x = 3$

$3 = x$ - coordinate of point of tangency, local maximum

$y = \pm\sqrt{6x - x^2}$

$x = 0$
$y = 0$

$x = 1 \quad y = \sqrt{5}$

$x = 4 \quad y = 8$

inc

$\frac{2}{\sqrt{5}}$

I	$f'(x)$
$-\infty, 3$	inc
$(3, \infty)$	dec

$\frac{-1}{8}$ c

$y = \pm\sqrt{6 - 1}$

D2

NO CALCULATOR ALLOWED

Work for problem 5(b)

(6, -4)

$\int y\,dy = \int 3-x\,dx$

$\frac{1}{2}y^2 =$

$\int y\,dy = \int 3-x\,dx$

$\frac{1}{2}y^2 + C = 3x - \frac{1}{2}x^2 + C$

$y = \sqrt{6x - x^2 + C}$

$-4 = \sqrt{36 - 36 + C}$

$-4 = \pm\sqrt{C}$

$g(x) = -\sqrt{6x - x^2 + 16}$