AP® Calculus BC
2002 Scoring Commentary

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Question 1

This problem presented a region bounded between two graphs and two vertical lines. Students were asked to use both integration and differentiation to answer some straightforward questions about this region. Part (a) required the use of a definite integral to find the area of the region. In part (b), the region is revolved about a horizontal line, resulting in a solid with cross sections in the shape of "washers," and the volume of the region was asked for, requiring another use of a definite integral. Students were expected to use the numerical integration capabilities of a graphing calculator to evaluate these definite integrals. Part (c) required the use of differentiation to find the absolute minimum height of the region.

The mean score was 5.57.

Sample A (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 2 points in part (a), 4 points in part (b), and 1 point in part (c). In part (c), the student set the derivative of $h(x)$ equal to 0, but did not find a critical point in the interval $\frac{1}{2} \le x \le 1$. This made the student ineligible for the answer point.

Question 2

This problem involved a "real-life" model of people entering and leaving an amusement park, and students were expected to interpret the meanings of their calculations in that context. In part (a), students needed to recognize that the function E represented the rate of accumulation of people entering the park, and hence that the total accumulation over a time interval could be obtained by a definite integral. Part (b) took the same idea a step further, asking students to calculate ticket revenues based on different pricing over two time intervals. Part (c) presented a function H defined in terms of a definite integral. The value of $H'(17)$ was easily computed, provided students recognized that the Fundamental Theorem of Calculus could be applied. More importantly, students were expected to interpret, in words, the meanings of both $H(17)$ and $H'(17)$. These interpretations played a role in part (d), which essentially asked students to find when the maximum value of the function H is achieved.

The mean score was 4.90.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 3 points in part (a), 0 points in part (b), 2 points in part (c), and 2 points in part (d). The student lost 1 point in part (b) since the function L, rather than the function E, was used in the second integral. In part (c), 1 point was lost since an incorrect value for $H'(17)$ was given.

Question 3

This problem, involving parametric equations, described the motion of a roller coaster car. Both coordinates of the car's position were given. While the components of the velocity vector could be calculated directly, these were also stated in the problem. The emphasis in this problem was on using the velocity vector's components in a variety of ways to describe characteristics of the car's motion and its path at various times. Part (a) asked for the slope of the car's path at time $t = 2,$ requiring students to formulate the evaluation of $\frac{dy}{dx}$ in terms of $\frac{dy}{dt}$ and $\frac{dx}{dt}.$ Part (b) asked students to first determine the time at which the car was at a specific horizontal position, a calculation that required the numerical equation solver of a graphing calculator. Once this time was determined, students needed to use the relationship between the velocity and its derivative to determine the components of the acceleration vector. Part (c) also required the use of a graphing calculator to determine the times at which $y'(t) = 0.$ At this instant, the speed of the car, given by the magnitude of the velocity vector, was $|x'(t)|.$ Students may or may not have used a graphing calculator to find the times at which $y(t) = 0$ in part (d). In either case, an appropriate definite integral expression needed to be given whose evaluation would result in the average speed of the car over the interval defined by these times.

The mean score was 3.04.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample D</u> (Score 7)

The student earned 7 points: 1 point in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). In part (d), the student earned the first point for finding the two correct values of t for which $y(t) = 0.$ The two subsequent points were lost because no definite integral for the average value of the speed was given.

Question 4

In this problem, students were given a graphical representation of a function *f*, and another function *g* that was defined in terms of a definite integral of *f*. While it was possible to find piecewise algebraic definitions for *f* and *g*, the questions asked were most efficiently and directly answered by using the Fundamental Theorem of Calculus and reasoning based on the graph of *f*. Part (a) asked for calculations of $g(-1)$, $g'(-1)$, and $g''(-1)$. These values could be found using, respectively, an area, ordinate, and slope related to the graph of *f*. Using the fact that $f = g'$, part (b) required relating the sign of *f* (positive or negative) to the behavior of *g* (increasing or decreasing). Similarly, using the fact that $f' = g''$, part (c) required relating the behavior of the slope of the graph of *f* to the concavity of the graph of *g*. Part (d) asked for a sketch of the graph of *g*. Utilizing previous parts of the problem helped in determining characteristics of the graph.

The mean score was 5.32.

<u>Sample B</u> (Score 9)

The student earned all 9 points.

<u>Sample C</u> (Score 7)

The student earned 7 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). The student had the incorrect sign in the calculation of $g(-1)$ in part (a), and the graph in part (d) was translated vertically.

Question 5

In this problem, a differential equation was presented. The first two parts of the problem involved slope fields and Euler's method, which are Calculus BC-only topics. The last two parts addressed topics common to Calculus AB and Calculus BC. Part (a) asked for two solution curves sketched against a slope field provided for the differential equation. Part (b) took one of the initial conditions from part (a) and asked for a demonstration of the use of Euler's method to approximate another point on the solution curve. In part (c), students were asked to find the y-intercept of a linear solution to the differential equation. One of the two solution curves sketched in part (a) should be a straight line, so there was a strong visual clue for students to use in checking the reasonableness of their answers. The slope field also provided strong visual clues for part (d), but students needed to provide more than a visual explanation to justify that the solution curve passing through the origin has a local maximum there. The second derivative test was the most straightforward way to justify this result.

The mean score was 4.00.

Sample B (Score 9)

The student earned all 9 points.

Sample D (Score 7)

The student earned 7 points: 2 points for part (a), 2 points for part (b), 1 point in part (c), and 2 points in part (d). In part (c), the student did not use the differential equation in the computation and lost the first point. In part (d), the justification that the function had a local maximum at the origin was not adequate.

Question 6

This problem presented students with an explicit Maclaurin series for a function f. In part (a), students were asked to determine its interval of convergence. Part (b) then asked students to derive a Maclaurin series for f' by manipulating the given series. Finally, in part (c), the evaluation of the Maclaurin series for f' at the specific value $x = -1/3$ resulted in a geometric series whose sum could be found by a simple calculation.

The mean score was 3.89.

Sample B (Score 9)

The student earned all 9 points.

Sample C (Score 7)

The student earned 7 points: 4 points in part (a), 2 points in part (b), and 1 point in part (c). In part (b), the student incorrectly claimed convergence at the right endpoint. In part (c), the student failed to evaluate the series at the given point.