AP® Calculus AB
2001 Scoring Commentary

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Question 1

This problem presented the student with two regions in the first quadrant.

Parts (a) and (b) of this problem required the student to use definite integrals to find the areas of two regions. Part (c) asked the student to find the volume of a solid with known cross sections (in this case, a solid of revolution). A calculator equation-solver (numerical or graphical) was needed to find the intersection of two curves that form part of the boundary of the regions in question. It was expected that students would also use the numerical integration capabilities of their calculators to evaluate the definite integrals.

The mean score was 4.84.

Sample A

The student earned all 9 points. Note that in part (a) the student used a correct *dy* integral.

Sample D

The student earned 7 points: 3 points in part (a), 3 points in part (b), and 1 point in part (c). In part (c) the student had an incorrect integrand and was ineligible for the answer point.

Question 2

This problem presented the student with tabular data representing water temperature readings in a pond recorded at regular time intervals.

Part (a) asked for an approximation of a derivative (with appropriate units) and part (b) asked for a (trapezoidal) approximation of a definite integral representing the average value of the temperature using this tabular data. In part (c), a function model for the water temperature was introduced. The student was asked to calculate a derivative analytically and to provide an interpretation of its meaning in this physical context. Part (d) required the student to calculate the average value of the function model over the time interval in part (b). This problem reflected the increased emphasis on working with multiple representations of functions. It illustrated two ways of working with data: approximations based on actual data values and analytic work based on a model approximating the data.

The mean score was 3.48.

<u>Sample A</u>

The student earned all 9 points.

<u>Sample C</u>

The student earned 7 points: 1 point in part (a), 2 points in part (b), 1 point in part (c), and 3 points in part (d). The second point in part (a) was not earned because units were omitted. The student did not earn the second point in part (c). The second point in part (c) required the student to use a word or phrase to indicate that the temperature is decreasing, and this student did not.

Question 3

This problem presented the student with a car's initial velocity and a piecewise linear graphical representation of the car's acceleration over a time interval. The different parts of the problem asked for interpretations and conclusions regarding the car's velocity. This required the student to both recognize the acceleration graph as that of the derivative of the velocity and to reason using this graphical representation of the derivative. It is possible that a student might have chosen to obtain a piecewise formula for the acceleration function in order to work analytically, but it would have been more efficient to reason directly from the graph.

Part (a) asked for an interpretation of the rate of change of the velocity at a given time (obtained by the sign of the value of the acceleration function). Parts (b), (c), and (d) each required calculating accumulated changes in velocity in terms of definite integrals of the acceleration function, all of which could be computed directly by summing the signed areas of triangles, rectangles and/or trapezoids. Part (c) also involved an analysis of local extrema and endpoint analysis in finding the time at which an absolute maximum velocity occurs. All parts of the problem asked for supporting reasons and justifications for the students' conclusions.

The mean score was 2.40.

<u>Sample A</u>

The student earned all 9 points.

<u>Sample D</u>

The student earned 7 points: 1 point in part (a), 2 points in part (b), 3 points in part (c), and 1 point in part (d). The fourth point in part (c) was not earned because the student did not properly eliminate $t = 18$ from consideration. The second point in part (d) was not earned because the student did not justify the answer provided.

Question 4

This problem presented the student with a derivative of a function *h* and an initial value of the function. Given this information, a student might have attempted to determine the function *h* explicitly by first antidifferentiating h' and then using the given initial condition to determine the constant of integration. However, the function *h* had a discontinuity at $x = 0$ and so the initial condition only determined the function for $x > 0$. For all parts of this problem the student could have used information supplied by the given derivative h' or its derivative h''.

Part (a) asked for the critical values and for an extrema analysis with justification. This justification could have involved either the first or second derivative tests. Part (b) asked for a concavity analysis that could be in terms of the second derivative or in terms of the increasing/decreasing behavior of the first derivative. Part (c) required the student to find the equation of the line tangent to the graph of *h* at the point defined by the initial condition. Part (d) tied parts (b) and (c) together by asking for a geometric analysis of the relationship of the line tangent to the graph of *h* found in part (c) by using the concavity information from part (b).

The mean score was 3.05.

Sample A

The student earned all 9 points.

Sample D

The student earned 7 points: 4 points in part (a), 2 points in part (b), and 1 point in part (c). The student correctly used the second derivative test in part (a) but failed to exclude "0" in part (b). The point in part (d) was not earned because the student did not make it clear that the curve was concave up on the given interval.

Question 5

This problem presented the student with a cubic polynomial function involving three undetermined coefficients, along with the location of a local minimum and a point of inflection of the graph.

Part (a) asked the student to determine the values of two of the coefficients using the given information. This problem was a slight variation of the routine exercise of finding the extrema and points of inflection of a given function. Part (b) provided additional information — the value of a definite integral of the cubic function — and the student was asked to use this to find the remaining coefficient of the polynomial.

The mean score was 6.11.

<u>Sample A</u>

The student earned all 9 points.

<u>Sample C</u>

The student earned 7 points: 3 in part (a) and 4 in part (b). In part (a) the student mistakenly substituted -1 into f'' and then followed with a calculation error in finding the value of b. The work in part (b) was consistent with the values of a and b obtained in part (a).

Question 6

This problem presented the student with a point on the graph of a function and an expression for the derivative of the function in terms of both x and y.

Part (a) asked the student to find the second derivative at the given point. Since the first derivative was given in terms of x and y, the second derivative could have been found using implicit differentiation. The information given in this problem also defined a differential equation. Part (b) asked the student to find an explicit formula for the function by solving this separable differential equation with initial condition. It is possible that a student might have first answered part (b) and then used this explicit solution to solve part (a) without using implicit differentiation.

The mean score was 3.64.

Sample A

The student earned all 9 points.

Sample C

The student earned 7 points: 2 points in part (a) and 5 points in part (b). The student correctly differentiated in part (a) but made an arithmetic error when evaluating at the point. The student correctly separated variables and found antiderivatives in part (b) but made an arithmetic error when solving for the constant of integration.