AP Calculus AB
1999 Free-Response Questions

These materials were produced by Educational Testing Service (ETS), which develops and administers the examinations of the Advanced Placement Program for the College Board. The College Board and Educational Testing Service (ETS) are dedicated to the principle of equal opportunity, and their programs, services, and employment policies are guided by that principle.

The College Board is a national nonprofit membership association dedicated to preparing, inspiring, and connecting students to college and opportunity. Founded in 1900, the association is composed of more than 3,900 schools, colleges, universities, and other educational organizations. Each year, the College Board serves over three million students and their parents, 22,000 high schools, and 3,500 colleges, through major programs and services in college admission, guidance, assessment, financial aid, enrollment, and teaching and learning. Among its best-known programs are the SAT®, the PSAT/NMSQT™, the Advanced Placement Program® (AP®), and Pacesetter®. The College Board is committed to the principles of equity and excellence, and that commitment is embodied in all of its programs, services, activities, and concerns.

1999

The College Board

Advanced Placement Examination

CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

REMEMBER TO SHOW YOUR SETUPS AS DESCRIBED IN THE GENERAL INSTRUCTIONS.

---

1. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

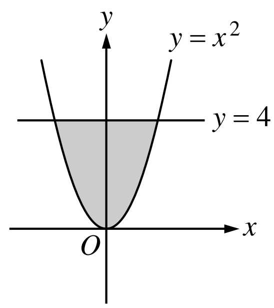
   (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

   (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

   (c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

   (d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

GO ON TO THE NEXT PAGE



2. The shaded region, R, is bounded by the graph of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.

(b) Find the volume of the solid generated by revolving R about the x-axis.

(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

GO ON TO THE NEXT PAGE

t (hours)	$R(t)$ (gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

3. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above shows the rate as measured every 3 hours for a 24-hour period.

   (a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

   (b) Is there some time t, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

   (c) The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

GO ON TO THE NEXT PAGE

4. Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all x.

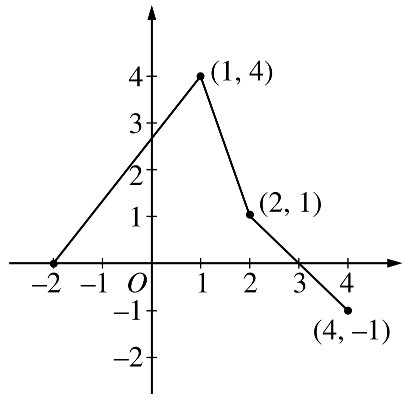
(a) Write an equation of the line tangent to the graph of f at the point where $x = 0$.

(b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does g have a local maximum at $x = 0$? Justify your answer.

GO ON TO THE NEXT PAGE



5. The graph of the function f, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)\,dt$.

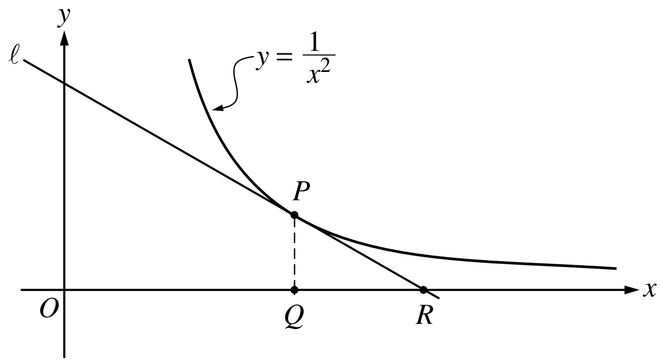
(a) Compute $g(4)$ and $g(-2)$.

(b) Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

(c) Find the absolute minimum value of g on the closed interval $[-2, 4]$. Justify your answer.

(d) The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

GO ON TO THE NEXT PAGE



6. In the figure above, line ℓ is tangent to the graph of $y = \frac{1}{x^2}$ at point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w, 0)$. Line ℓ crosses the x-axis at point R, with coordinates $(k, 0)$.

   (a) Find the value of k when $w = 3$.

   (b) For all $w > 0$, find k in terms of w.

   (c) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of k with respect to time?

   (d) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change of the area of $\triangle PQR$ with respect to time? Determine whether the area is increasing or decreasing at this instant.

END OF EXAMINATION