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2. Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

<table>
<thead>
<tr>
<th>Number who actually show up</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
<th>40</th>
<th>41</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.46</td>
<td>0.30</td>
<td>0.16</td>
<td>0.05</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Assume that 41 tickets are sold for each flight.

(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?

A group of 36, 37, or 38 passengers can get a seat.

\[ 0.46 + 0.30 + 0.16 = 0.92 \]

The probability that all passengers who show up for this flight will get a seat is 0.92.

(b) What is the expected number of no-shows for this flight?

The data given above can be interpreted as the table shown left:

<table>
<thead>
<tr>
<th>Number who actually did not show up</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
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<td>0.30</td>
<td>0.16</td>
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<td>0.01</td>
</tr>
</tbody>
</table>

\[ E(X) = \sum np = 5 \times 0.46 + 4 \times 0.30 + 3 \times 0.16 + 2 \times 0.05 + 1 \times 0.02 + 0 \times 0.01 \]

\[ = 4.1 \]

The expected number of no-shows for this flight is 4.1.

(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?

If a group of 36 or 37 passengers show up, not all passenger seats are filled on a flight.

\[ P(\text{not all seats filled}) = 0.46 + 0.30 = 0.76 \]

\[ P(36 \text{ passengers}) = 0.46 \]

\[ P(36 \text{ passengers} \mid \text{not all seats filled}) = \frac{P(36 \text{ passengers} \text{ and not all seats filled})}{P(\text{not all seats filled})} = \frac{0.46}{0.76} = 0.61 \]

There is 0.61 probability that only 36 passengers showed up for the flight, given that not all passenger seats are filled on a flight.

GO ON TO THE NEXT PAGE.
2. Airlines routinely overbook flights because they expect a certain number of no-shows. An airline runs a 5 P.M. commuter flight from Washington, D.C., to New York City on a plane that holds 38 passengers. Past experience has shown that if 41 tickets are sold for the flight, then the probability distribution for the number who actually show up for the flight is as shown in the table below.

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</table>

Assume that 41 tickets are sold for each flight.

(a) There are 38 passenger seats on the flight. What is the probability that all passengers who show up for this flight will get a seat?

\[
P(\text{at most 38 passengers show up}) = 0.46 + 0.30 + 0.16 = 0.92
\]

(b) What is the expected number of no-shows for this flight?

\[
\begin{align*}
\text{No shows} & \quad 5 & 4 & 3 & 2 & 1 & 0 \\
\text{Probability} & \quad 0.46 & 0.30 & 0.16 & 0.05 & 0.02 & 0.01 \\
\end{align*}
\]

\[
0.46(5) + 0.30(4) + 0.16(3) + 0.05(2) + 0.02(1) + 0.01(0) = 2.3 + 1.2 + 0.48 + 0.1 + 0.2 = 4.1
\]

(c) Given that not all passenger seats are filled on a flight, what is the probability that only 36 passengers showed up for the flight?

\[
P(36 \text{ shows up}) \rightarrow P(36 \text{ shows up } | 36 \leq X < 38)
\]

\[
P(36 \leq X < 38) = \frac{0.46 \times (0.46 + 0.30)}{(0.46 + 0.30)}
\]

\[
= \frac{0.46 \times 0.76}{1.76} = 0.446
\]