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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Point of intersection
\( y(n) = g(n) \)
\( 1 + \sin(2n) = e^{2n} \)
\( 1 + \sin(2n) - e^{2n} \)
\( n = 1.1357 \)

Area of \( R \)
\[
\begin{align*}
\text{Area of } R &= \int_{0}^{1.1357} [f(x) - g(x)] \, dx \\
&= \int_{0}^{1.1357} [1 + \sin(2x) - e^{2x}] \, dx \\
&= 0.4291
\end{align*}
\]

G.C. \( = 0.4291 \)

Continue problem 1 on page 5.
Work for problem 1(b)

\[ \text{Volume} = \pi \int_{0}^{\frac{\pi}{2}} (1 + \sin 2\theta)^{2} d\theta \]

\[ = \pi \int_{0}^{\frac{\pi}{2}} (1 + \sin 2\theta)^{2} (1) d\theta \]

\[ = \pi \int_{0}^{\frac{\pi}{2}} (1 + 2\sin \theta \cos \theta)^{2} d\theta \]

\[ = \pi \int_{0}^{\frac{\pi}{2}} (1 + 2\sin \theta \cos \theta)^{2} d\theta \]

\[ \approx \pi \cdot 3.581 \]

\[ \# \]

Work for problem 1(c)

\[ k = \frac{f(\pi) - g(\pi)}{2} \]

\[ = \frac{1 + \sin(2\pi) - e^{-\pi/2}}{2} \]

\[ \approx 0.0247 \]

\[ \# \]

\[ \text{Volume} = \frac{1}{2} \pi \int_{0}^{\frac{\pi}{2}} \left( \frac{1 + \sin(2\pi) - e^{-\pi/2}}{2} \right)^{2} d\theta \]

\[ \approx 0.0247 \pi \]

\[ \# \]

GO ON TO THE NEXT PAGE.
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ A = \int_{a}^{b} f(x) - g(x) \, dx \]
\[ = \int_{0}^{1.1557} \left( 1 + \sin(2x) - e^{\frac{x}{2}} \right) \, dx \]
\[ = 0.429 \text{ units}^2 \]
Work for problem 1(b)

\[ V = \pi \int_{a}^{b} (R^2 - r^2) \, dx \]
\[ = \pi \int_{0}^{1.1367} \left( (1+\sin(2x) - e^{x/2})^2 - (e^{x/2})^2 \right) \, dx \]
\[ = 3.9398 \text{ units}^3 \]

Work for problem 1(c)

\[ V = \int_{a}^{b} A(x) \, dx \]
\[ r = \frac{d}{z} \]

\[ A_{\text{semicircle}} = \frac{1}{2} \pi r^2 \]
\[ = \frac{1}{2} \pi \left( \frac{d^2}{4} \right) \]
\[ = \int_{0}^{1.1367} \left( (1+\sin(2x) - e^{x/2})^2 \cdot \frac{1}{2} \pi \right) \, dx \]
\[ = 0.169 \text{ units}^3 \]

GO ON TO THE NEXT PAGE.
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ 1 + \sin(2x) = e^{\frac{x}{2}} \]

\[ x = 0.136, 0.01136 \]

\[ \int_0^1 (1 + \sin(2x) - e^{\frac{x}{2}}) \, dx = \text{Using calculator} \rightarrow 0.029 \]

Continue problem 1 on page 5.
Work for problem 1(b)

\[
\pi \int_0^{\frac{\pi}{2}} \left( 1 + \sin(2x) - e^{\frac{x^2}{2}} \right)^2 \, dx \approx 0.621
\]

---

Work for problem 1(c)

\[
\int_0^{\frac{\pi}{2}} \left( 1 + \sin(2x) - e^{\frac{x^2}{2}} \right)^2 \, dx
\]
Work for problem 2(a)

\[ W(15) = 131.782 \text{ gal/hour (water pumped into)} \]
\[ R(15) = 252.872 \text{ gal/hour (water removed)} \]

Therefore, the amount of water is decreasing because \( R(t) > W(t) \) when \( t=15 \), meaning that water is being removed at a higher rate than is water being pumped into the tank.

Work for problem 2(b)

\[
1200 + \int_0^{18} (W(t) - R(t)) \, dt = \text{total gallons of water in the tank at } t=18
\]

\[ = 1200 + 109.788 \]

\[ = 1309.79 \]

\[ \approx 1310 \text{ gallons of water.} \]

Continue problem 2 on page 7.
Work for problem 2(c)

\[ W(t) - R(t) = 0 \rightarrow \text{indicates a max or min.} \]

\[ t = 0, 6.49484, 12.9748 \]

End points: 0, 18.

\[ 1200 + \int_0^{6.49484} (W(t) - R(t)) \, dt = 525 \text{ gallons} \]

\[ 1200 + \int_6^{12.9748} (W(t) - R(t)) \, dt = 1697.44 \text{ gallons} \]

\[ 1200 + \int_0^{18} (W(t) - R(t)) \, dt = 1310 \text{ gallons} \]

Therefore, the amount of water reaches an absolute minimum when \( t = 6.49484 \).

Work for problem 2(d)

\[ \rightarrow \text{the amount of water, in gallons, when } t = 18 \]

\[ 1310 - \int_{18}^{t} R(t) \, dt = 0 \]
Work for problem 2(a)

No, because the water is being removed at a greater rate than it is being pumped in.

Work for problem 2(b)

\[1200 \text{ gallons in} - \text{ gallons out}\]

\[
in: \int_0^{18} 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) dt \approx 2695.46 \text{ gallons}
\]

\[
out: \int_0^{18} 275 \sin^2\left(\frac{t}{3}\right) dt \approx 2585.67 \text{ gallons}
\]

\[\approx 1200 + 2695.46 - 2585.67 \approx 1309.79 \text{ gallons}
\]

\[\approx 1310 \text{ gallons}\]

Continue problem 2 on page 7.
Work for problem 2(c)

\[ 1200 + \int_{0}^{6.49484} W(t) - \int_{0}^{6.49484} R(t) \]

\[ = 525.242 \text{ gallons} \]

at time \( t = 6.49484 \), the greatest amount of water has been removed to the amount pumped in; up to \( t = 6.49484 \), the water level is decreasing.

Work for problem 2(d)

\[ \int_{18}^{k} 275 \sin^2 \left( \frac{t}{3} \right) dt = 1310 \text{ gal.} \]

(the amount removed from time \( t = 18 \) to some time \( k \) is approximately 1310 gallons)

-7-

GO ON TO THE NEXT PAGE.
Work for problem 2(a)

\[ S(t) = 95 \sqrt{t} \sin^2 \left( \frac{1}{6} t \right) - 275 \sin^2 \left( \frac{1}{3} t \right) \]

\[ S(15) = 95 \sqrt{15} \sin^2 \left( \frac{15}{6} \right) - 275 \sin^2 \left( \frac{1}{3} (15) \right) = -121.09 \text{ gallons} \]

So, no water inlet increasing but decreasing

Work for problem 2(b)

\[ \int_{0}^{18} (95 \sqrt{t} \sin^2 \left( \frac{1}{6} t \right) - 275 \sin^2 \left( \frac{1}{3} t \right)) \, dt = 109.79 \text{ gallons} \]

Continue problem 2 on page 7.
Work for problem 2(c)

\[ 9s \sqrt{E} \sin^2 \left( \frac{1}{6} t \right) - 275 \sin^2 \left( \frac{1}{3} t \right) = 0 \]

\[ 9s \sqrt{E} \sin^2 \left( \frac{1}{6} t \right) = 275 \sin^2 \left( \frac{1}{3} t \right) \]

\[ \frac{\sin^2 \left( \frac{1}{6} t \right)}{\sin^2 \left( \frac{1}{3} t \right)} = \frac{275}{9s \sqrt{E}} \]

\[ \frac{s \sin^2 \left( \frac{1}{6} t \right)}{\sin^2 \left( \frac{1}{3} t \right)} = \frac{275}{9s \sqrt{E}} \]

Work for problem 2(d)

\[ \int_{18}^{K} 275 \sin^2 \left( \frac{1}{3} t \right) dt = 0 \]
Work for problem 3(a)

a) \[ a(t) = (v(t))' = (\ln (t^2 - 3t + 3))' \]
   \[ a(u) = (\ln (t^2 - 3t + 3))' \bigg|_{t=u} = 0, 7.14 \]

Answer: \[ a(u) = 0, 7.14 \]

Work for problem 3(b)

b) Particle changes direction \( \iff v(t) \) changes sign.
   \[ \Rightarrow v(t) = 0 \iff \ln (t^2 - 3t + 3) = 0 \]

Particle changes direction at \( t = 1 \) and \( t = 2 \)

The particle is moving leftwards at time \( 1 < t < 2 \).

\[
\begin{align*}
  v(t) & \quad + \quad - \quad + \quad + \\
  0 & \quad 1 & \quad 2 & \quad 5 & \quad t
\end{align*}
\]

Continue problem 3 on page 9.
Work for problem 3(c)

c) \[ x(t) = \int_0^t (v(t)) \, dt + x(0) \]

\[ x(2) = \int_0^2 (\ln(x^2 - 3x + 3)) \, dt + 8 \]

\[ x(2) = 8,3686 \]

Answer: \[ x(2) = 8,368 \]

Work for problem 3(d)

d) Speed = \[ |v(t)| \]

Average speed over the interval \(0 \leq t \leq 2\):

\[ = \frac{1}{2} \int_0^2 |v(t)| \, dt = \frac{1}{2} \int_0^2 (|\ln(t^2 - 3t + 3)|) \, dt \]

Average speed = 0,370509.

Answer: 0,371.
Work for problem 3(a)

\[ 0 \leq t \leq 5 \]

\[ v(t) = \ln(t^2 - 3t + 3) \]

\[ x = 5 \quad t = 0 \]

\[ a(t) = v'(t) \]

\[ a(4) = v'(4) \]

\[ a(4) = .714 \]

---

Work for problem 3(b)

Particle changes direction when

\[ v(t) \] changes from +ve \( \rightarrow \) -ve or -ve \( \rightarrow \) +ve

\[ s(4) > 0 \quad \text{and} \quad v(4) < 0 \]

\[ s(4) > 0 \quad \text{and} \quad v(4) < 0 \]

So when \( v(t) = 0 \)

\[ \ln(t^2 - 3t + 3) = 0 \]

\[ t = 1, 2 \]

The particle changes direction when \( t = 1, 2 \).
Work for problem 3(c)

\[ t = 2 \]
\[ S(t) = \text{?} \]
\[ \int v(t) \, dt = S(t) \]
\[ S(2) = \int_0^2 \ln(t^2 - 3t + 3) \, dt \]
\[ S(2) = 0.369 \]

At \( t = 0 \), \( x = 8 \)

\[ S_0 + 8 = 8 \cdot 369 \]

---

Work for problem 3(d)

\[ \text{Avg speed} = \frac{1}{2-0} \int_0^2 \ln(t^2 - 3t + 3) \, dt \]

\[ = 0.184 \]

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 3(a)

\[ v(t) = \ln\left( t^2 - 3t + 3 \right), \quad 0 \leq t \leq 5 \]

\[ s(t) = v'(t) = \frac{1}{t^2 - 3t + 3} \cdot (2t - 3) = \frac{2t - 3}{t^2 - 3t + 3} \]

\[ a(t) = \frac{5}{16 - 12t} = \frac{5}{4} \]

---

Work for problem 3(b)

A particle changes direction when

\[ v(t) = \ln\left( t^2 - 3t + 3 \right) = 0 \]

by calculating:

\[ t = 2, \quad t = 1 \]

In time, \( t = 1 \), the particle changes its direction.

Particle moves to the left in the interval between \( t = 1 \) and \( t = 2 \), i.e., to \((1, 2)\)

Continue problem 3 on page 9.
Work for problem 3(c)

\[ s(t) = \text{the position of particle} \]

\[ t = \int v(t) \, dt = \int \ln \left( t^2 - st + s \right) \, dt = -5 \ln \left( 1 - \frac{2 \, \tan^{-1} \left( \frac{\sqrt{3} \left( t + s \right)}{3} \right)}{2} \right) + \ln(t - s) \]

\[ 2t + \frac{\sqrt{3} \, \tan^{-1} \left( \frac{\sqrt{3} \left( t + s \right)}{3} \right)}{180} \]

\[ s(0) = -3 \ln \frac{1}{2} + 2 \ln 1 - 9 + \frac{\sqrt{3} \, \tan^{-1} \left( \frac{\sqrt{3}}{3} \right)}{180} = \frac{58.5 \cdot 30}{180} - 9 = \frac{\sqrt{3} \, \tan^{-1} \left( \frac{\sqrt{3}}{3} \right)}{6} \]

\[ s(2) = \frac{-3 \ln 1}{2} + 2 \ln 1 - 9 + \frac{\sqrt{3} \, \tan^{-1} \left( \frac{\sqrt{3}}{3} \right)}{180} = \frac{58.5 \cdot 30}{180} - 9 = -3.09 \]

\[ s(0) = -3 \ln 3 + \frac{\sqrt{3} \, \tan^{-1} \left( \frac{\sqrt{3}}{3} \right)}{2} \]

Work for problem 3(d)

\[ \text{speed} = \left| \text{velocity} \right| \]

Average speed = \[ \left| \frac{s(2) - s(0)}{2 - 0} \right| = \left| \frac{-3.09 - 5(0)}{2} \right| = \left| \frac{-3.09}{2} \right| = 0.86 \]

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 4(a)

\[ g(-1) = \int_{-4}^{-1} f(t) \, dt \]

\[ = \frac{-3}{2} + \frac{(2)}{3} \cdot (3) \]

\[ = \frac{-15}{2} \]

\[ g'(x) = f(-1) \]

\[ = -2 \]

\[ g''(x) = f'(-1) \]

\[ = g''(-1) \]

\[ g''(-1) \text{ does not exist} \]
Work for problem 4(b)

\[ f(x) \text{ has an inflection pt when } x = 1 \]

\[ f''(x) = f'(x) \]

Inflection pt occurs when \( f'(x) \) goes from + to - or from - to + (concavity change). This happens when \( x = 1 \).

Work for problem 4(c)

\[ F(t) = \int_{-3}^{t} f(t) \, dt \]

\[ F(2) \text{ is 0 when } x = 3, 1, -1 \]

\[ \int_{-3}^{3} f(t) \, dt, \int_{1}^{3} f(t) \, dt, \int_{-3}^{1} f(t) \, dt \text{ are all zero.} \]

Work for problem 4(d)

\[ h(x) \text{ decreases when } 0 < x < 2 \]

\[ h'(x) = -f(t) \] \( (h(x) = \int_{-3}^{x} f(t) \, dt) \)

\[ h'(x) < 0 \text{ when } f(t) > 0 \]

\[ f(t) > 0 \text{ when } 0 < x < 2 \]

GO ON TO THE NEXT PAGE.
Work for problem 4(a)

\[ g(-1) = \frac{-7}{2} - 6 = -7.5 \]

\[ g'(-1) = f(-1) = -2 \]

\[ g''(-1) \text{ does not exist, since } f'(-1) \text{ does not exist} \]

\( f(x) \) is not differentiable at point \( x = -1 \)

Continue problem 4 on page 11.
Work for problem 4(b)

\[ g''(x) = f'(x) \]
\[ g'(x) = 0 \Rightarrow f'(x) = 0 \]

but there are no points where \( f'(x) = 0 \) on the interval \((-4, 3)\) \Rightarrow there are no points of inflection of function \( g(x) \) on the same interval \((-4, 3)\)

Work for problem 4(c)

There is only one value of \( x \) \Rightarrow

\[ x = 1, \quad x = -1 \]

\[ h(x) = \frac{3}{2} \int x f(t) \, dt \]

\[ h(1) = 1 - 1 = 0, \quad h(-1) = 1 - 1 + 1 - 1 = 0 \]

Work for problem 4(d)

\[ h'(x) = \frac{d}{dx} \left( \frac{3}{2} \int x f(t) \, dt \right)' = -f(x) \]

\[ h'(x) < 0 \text{ for } h(x) \text{ to decrease } \Rightarrow f(x) > 0 \Rightarrow x \in [0, 2] \]
Work for problem 4(a)

\[ g(-1) = \int_{-4}^{-1} f(t) \, dt = 7.5 \]

\[ g'(-1) = -2 \]

\[ g''(-1) = \text{does not exist} \]
Work for problem 4(b)

$q(x)$ experiences a point of inflection where $q''(x) = 0 \quad f''(x) = 0$, hence it is where $f(t)$ has critical points. $X = 1$

Work for problem 4(c)

$x = -1$, where $h(x) = 0$

Work for problem 4(d)

where $w(x)$ = negative, hence $(-4, -1)(-1, 0)(2, 3)$. 

GO ON TO THE NEXT PAGE.
Work for problem 5(a)

\[ 2y \frac{dy}{dx} = y + x \frac{dy}{dx} \]

\[ (2y - x) \frac{dy}{dx} = y \]

\[ \frac{dy}{dx} = \frac{y}{2y - x} \]

Work for problem 5(b)

\[ \frac{dy}{dx} = \text{slope of the tangent} \]

\[ \frac{1}{2} = \frac{y}{2y - x} \]

\[ 2y = 2y - x \implies x = 0 \]

\[ x = 0 \implies y^2 = 2 \implies y = \pm \sqrt{2} \implies -\sqrt{2} \]

\( (0, \sqrt{2}) \) and \( (0, -\sqrt{2}) \) are points on the curve where the tangent has slope \( \frac{1}{2} \).

Continue problem 5 on page 13.
Work for problem 5(c)

The line tangent is horizontal $\Rightarrow$ slope of tangent is zero $\Rightarrow \frac{dy}{dx} = 0$.

If $\frac{dy}{dx} = 0$, then $y = 0$.

Substituting $y = 0$ in $y^2 = 2 + xy$ gives $0 = 2$ which is false.

$\therefore \frac{dy}{dx}$ cannot be zero $\Rightarrow$ there is no point $(x, y)$ where the line tangent to the curve is horizontal.

Work for problem 5(d)

$\frac{2y}{dt} \frac{dy}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x$

$y = 3 \Rightarrow 9 = 2 + 3x \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$

Or $+5$:

$2(3)(6) = \frac{dx}{dt} (3) + (6) \left( \frac{7}{3} \right)$

$36 = 3 \frac{dx}{dt} + 14$

$\frac{dx}{dt} = \frac{36 - 14}{3}$

$\frac{dx}{dt} = \frac{22}{3}$

GO ON TO THE NEXT PAGE.
Work for problem 5(a)

\[
\begin{align*}
y^2 &= 2 + xy \\
2y \frac{dy}{dx} &= \frac{dx}{dx}y + x \frac{dy}{dx} \\
2y \frac{dy}{dx} &= y + x \frac{dy}{dx} \\
2y \frac{dy}{dx} - x \frac{dy}{dx} &= y \\
\frac{dy}{dx} (2y - x) &= y \\
\frac{dy}{dx} &= \frac{y}{2y - x} \\
\end{align*}
\]

Work for problem 5(b)

\[
(x_1, y_1) \quad y - y_1 = m(x - x_1)
\]

\[
\begin{align*}
y &= \frac{1}{2} \\
2y &= 2y - x \\
y &= y - \frac{1}{2}x \\
y + \frac{1}{2}x &= \frac{y}{2} \\
y^2 &= 2 + 0(y) \\
y^2 &= 2 + 0 \\
y &= \pm \sqrt{2} \\
x &= 0 \\
(0, -\sqrt{2}) \\
(0, \sqrt{2})
\end{align*}
\]

Continue problem 5 on page 13.
Work for problem 5(c)

\[ f'(c) = 0 \]

\[ \frac{y}{2y-x} = 0 \]

\[ y = 0 \]

\[ 2y-x = 0 \]

\[ \frac{0}{x} = 0 \]

\[ y = 0 \text{ or } x = 0 \]

Can't solve for \( x \) or \( y \).

Work for problem 5(d)

\[ y^2 = 2 + xy \]

\[ 2y \frac{dy}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x \]

\[ 2y \frac{dy}{dt} - \frac{dy}{dt} x = \frac{dx}{dt} \]

\[ \frac{2(3)(6) - 6(5)}{3} = \frac{dx}{dt} \]

\[ \frac{36 - 30}{3} = \frac{dx}{dt} \]

\[ \frac{6}{3} = \frac{dx}{dt} = 2 \]

GO ON TO THE NEXT PAGE.
Work for problem 5(a)

\[ y^2 = 2 + xy \]

\[ 2y \frac{dy}{dx} = x \frac{dy}{dx} + y \]

\[ (2y-x) \frac{dy}{dx} = y \]

\[ \frac{dy}{dx} = \frac{y}{2y-x} \]

Work for problem 5(b)

\[ \frac{1}{2} = \frac{y}{2y-x} \]

\[ -x = 2y \]

\[ x = 0 \]

\[ x = 0, y = \sqrt{2} \]

\[ 0, \sqrt{2} \]

Continue problem 5 on page 13.
Work for problem 5(c)

\[
\frac{dy}{dx} = \frac{y}{2y-x}
\]

\[
y^2 = 2 + xy
\]

\[
\therefore 2y \neq x
\]

Work for problem 5(d)

\[
y^2 = 2 + xy
\]

\[
2y \frac{dy}{dx} = x \frac{dy}{dx} + y \frac{dx}{dx}
\]
Work for problem 6(a)

\[
\frac{dy}{dx} = \frac{-xy^2}{2}
\]

\[
\frac{- (2)(1)}{2}
\]

\[
\frac{- (2)(4)}{2}
\]

\[
\frac{- (-1)(1)}{2} = \frac{- (-1)(4)}{2}
\]

Work for problem 6(b)

\[f(-1) = 2\]

\[
\frac{dy}{dx} = \frac{-xy^2}{2} = \frac{-(-1)(4)}{2} = 2
\]

\[y = m(x-x_1) + y_1\]

\[y = 2(x - (-1)) + 2\]

\[y = 2x + 4\]

Continue problem 6 on page 15.
Work for problem 6(c)

\[
\frac{dy}{dx} = -\frac{xy^2}{2}
\]

\[2\, dy = -x\, y^2 \, dx\]

\[\frac{2}{y^2} \, dy = x \, dx\]

\[-2\left(y^{-2}\right) \, dy = \int x \, dx\]

\[2\, y^{-1} = \frac{1}{2} x^2 + C\]

\[-1 = 2\, y\]

\[2\left(2^{-1}\right) = \frac{1}{2} (-1)^2 + C\]

\[1 = \frac{1}{2} + C\]

\[C = \frac{1}{2}\]

\[2\, y^{-1} = \frac{1}{2} x^2 + \frac{1}{2}\]

\[\frac{1}{y} = \frac{1}{4} x^2 + \frac{1}{4}\]

\[\frac{1}{y} = \frac{x^2 + 1}{4}\]

\[y = \frac{4}{x^2 + 1}\]

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

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- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.
Work for problem 6(a)

Work for problem 6(b)

\[ y - 2 = m(x - 1) \]
\[ y - 2 = 2(x - 1) \]
\[ y = 2x \]

Continue problem 6 on page 15.
Work for problem 6(c)

\[ \int \frac{dy}{y^2} = \int \frac{-x \, dx}{2} \]

\[ \ln|y^2| = -\frac{x^2}{4} + C \]

\[ \ln 4 = -\frac{1}{4} + C \]

\[ 4 = Ce^{-\frac{1}{4}} \]

\[ \frac{4}{e^{-\frac{1}{4}}} = C \]

\[ y^2 = Ce^{-\frac{x^2}{4}} \]

\[ y = \sqrt{Ce^{-\frac{x^2}{4}}} \]

END OF EXAM

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- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.
Work for problem 6(a)

Work for problem 6(b)

\[ (-1, 2) \]

\[
\frac{dy}{dx} = \frac{1 \times 4}{2} = 2
\]

\[ y = 2(x+1) + 2 \]

Continue problem 6 on page 15.
Work for problem 6(c)

\[
\frac{dy}{dx} = \frac{-xy^2}{2}
\]

\[2\,dy = -xy^2\,dx\]

\[ay = -\frac{1}{3x}y^3\]

\[= -\frac{1}{3}xy^3 + c\]

\[(-1, 2)\]

\[4 = \frac{8}{3} + c\]

\[c = \frac{12}{3} - \frac{8}{3}\]

\[= \frac{4}{3}\]

\[2\,y = -\frac{1}{3}xy^3 + \frac{4}{3}\]

\[y = -\frac{1}{6}xy^3 + \frac{4}{6}\]

END OF EXAM