



AP[®] Calculus AB 2005 Sample Student Responses Form B

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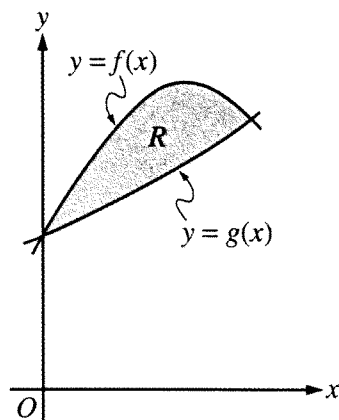
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CALCULUS AB
SECTION II, Part A
 Time—45 minutes
 Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

point of intersection

$$f(x) = g(x)$$

$$1 + \sin(2x) = e^{x/2}$$

$$1 + \sin(2x) - e^{x/2}$$

$$x = 1.1357$$

$$\text{Area of } R = \int_0^{1.1357} [f(x) - g(x)] dx$$

$$= \int_0^{1.1357} 1 + \sin(2x) - e^{x/2} dx$$

$$\text{G.C.} = 0.4291$$

Continue problem 1 on page 5.

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Work for problem 1(b)

$$\begin{aligned} \text{Volume} &= \pi \int_0^{1.1357} R^2 - r^2 \, dx \\ &= \pi \int_0^{1.1357} (1 + \sin 2x)^2 - (e^{x/2})^2 \, dx \end{aligned}$$

G.C. = 1.3581 π
~~#~~

Work for problem 1(c)

$$\begin{aligned} \text{radius} &= \frac{f(x) - g(x)}{2} \\ &= \frac{1 + \sin(2x) - e^{x/2}}{2} \end{aligned}$$

$$\begin{aligned} \text{Volume} &= \frac{1}{2} \pi \int_0^{1.1357} \left(\frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 dx \\ &= 0.0247 \pi \\ &\quad \text{---\#} \end{aligned}$$

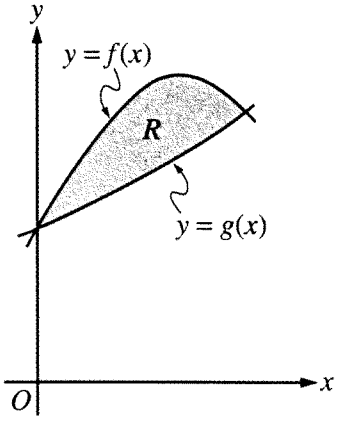
area of cross section.

$$\begin{aligned} &= \frac{1}{2} \pi r^2 \\ &= \frac{1}{2} \times \pi \times \left(\frac{1 + \sin(2x) - e^{x/2}}{2} \right)^2 \end{aligned}$$

GO ON TO THE NEXT PAGE.

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$\begin{aligned}
 A &= \int_a^b f(x) - g(x) dx \\
 &= \int_0^{1.1357} (1 + \sin(2x) - e^{x/2}) dx \\
 &= \boxed{0.429 \text{ units}^2}
 \end{aligned}$$

Continue problem 1 on page 5.

Work for problem 1(b)

$$\begin{aligned}
 V &= \pi \int_a^b (R^2 - r^2) dx \\
 &= \pi \int_0^{1.1357} \left((1 + \sin(2x) - e^{x/2})^2 - (e^{x/2})^2 \right) dx \\
 &= \boxed{3.9398 \text{ units}^3}
 \end{aligned}$$

Work for problem 1(c)

$$\begin{aligned}
 V &= \int_a^b A(x) dx & r &= \frac{d}{2} \\
 &= \int_0^{1.1357} \left(\frac{(1 + \sin(2x) - e^{x/2})^2}{4} \cdot \frac{1}{2} \pi \right) dx \\
 &= \boxed{0.169 \text{ units}^3}
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{semicircle}} &= \frac{1}{2} \pi r^2 \\
 &= \frac{1}{2} \pi \left(\frac{d^2}{4} \right)
 \end{aligned}$$

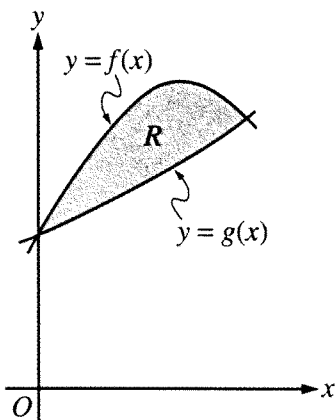
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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



Work for problem 1(a)

$$1 + \sin(2x) = e^{\frac{x}{2}}$$

1.136

$$x = 0, 1.336, 0, 1.36$$

$$\int_0^{1.136} (1 + \sin(2x) - e^{\frac{x}{2}}) dx \Rightarrow \text{using calculator} \rightarrow \boxed{0.429}$$

Continue problem 1 on page 5.

C2

1 1 1 1 1 1 1 1 1 1

Work for problem 1(b)

$$= \pi \int_0^{1.136} (1 + \sin(2x) - e^{\frac{x}{2}})^2 dx \rightarrow 0.621$$

Work for problem 1(c)

$$\frac{1}{2} \pi \int_0^{1.136} (1 + \sin(2x) - e^{\frac{x}{2}})^2 dx.$$

GO ON TO THE NEXT PAGE.

□

Work for problem 2(a)

$$W(15) = 131.782 \text{ gal/hour (water pumped into)}$$

$$R(15) = 252.872 \text{ gal/hour (water removed)}$$

Therefore, the amount of water is decreasing b/c $R(t) > W(t)$ when $t=15$, meaning that water is being removed at a higher rate than is water being pumped into the tank.

Work for problem 2(b)

$$1200 + \int_0^{18} (W(t) - R(t)) dt = \text{total gallons of water in the tank at } t=18$$

$$= 1200 + 109.788$$

$$= 1309.79$$

≈ 1310 gallons of water.

Continue problem 2 on page 7.

Work for problem 2(c)

$W(t) - R(t) = 0 \rightarrow$ indicates a max or min.

$$t = 0, 6.49484, 12.9748$$

End points: 0, 18.

$$1200 + \int_0^{6.49484} (W(t) - R(t)) dt = \boxed{525 \text{ gallons}}$$

$$1200 + \int_0^{12.9748} (W(t) - R(t)) dt = 1697.44 \text{ gallons}$$

$$1200 + \int_0^0 (W(t) - R(t)) dt = 1200 \text{ gallons}$$

$$1200 + \int_0^{18} (W(t) - R(t)) dt = 1310 \text{ gallons.}$$

Therefore, the amount of water reaches an absolute minimum when $t = 6.49484$.

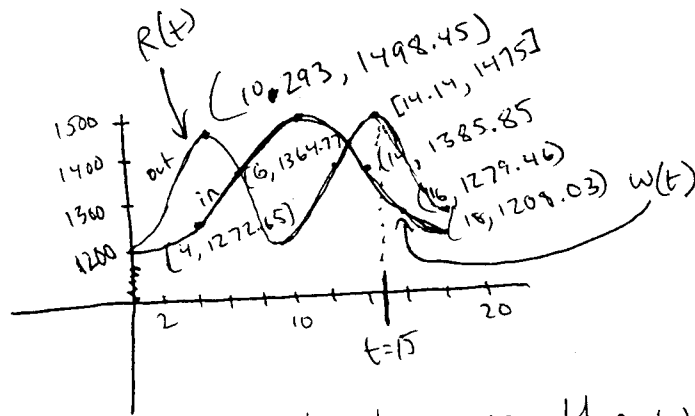
Work for problem 2(d)

\rightarrow the amount of water, in gallons, when $t = 18$.

$$1310 - \int_{18}^k R(t) dt = 0$$

GO ON TO THE NEXT PAGE.

Work for problem 2(a)



No, because the water is being removed at a greater rate than it is being pumped in.

Work for problem 2(b)

1200 + gallons in - gallons out

in: $\int_0^{18} 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) dt \approx 2695.46$ gallons

out: $\int_0^{18} 275 \sin^2\left(\frac{t}{3}\right) dt \approx 2585.67$ gallons

$\approx 1200 + 2695.46 - 2585.67 \approx$

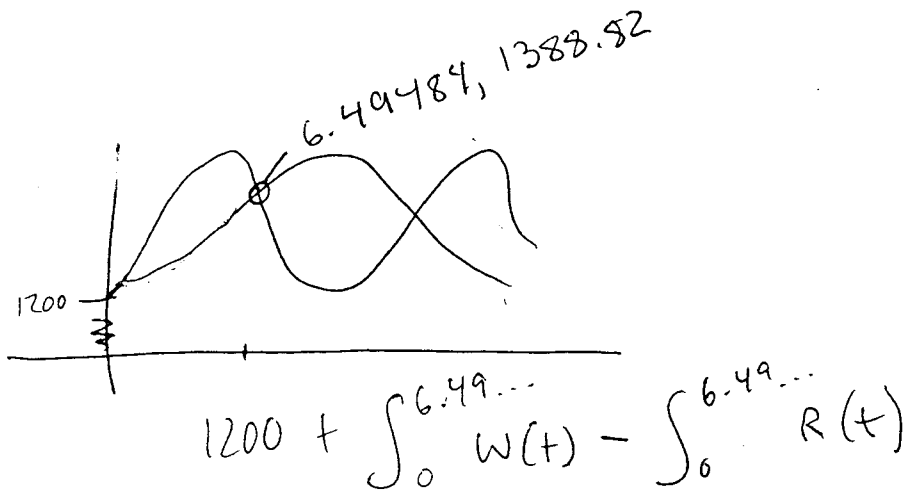
1309.79 gallons

$\approx \boxed{1310 \text{ gallons}}$

Continue problem 2 on page 7.

2 2 2 2 2 2 2 2 2 2

Work for problem 2(c)



$$= 525.242 \text{ gallons}$$

at time $t = 6.49484$, the greatest amount of water has been removed to the amount pumped in; up to $t = 6.49484$, the water level is decreasing.

Work for problem 2(d)

$$\int_{18}^k 275 \sin^2(t/3) dt = 1310 \text{ gal.}$$

(the amount removed from time $t = 18$ to some time k is approximately 1310 gallons)

GO ON TO THE NEXT PAGE.

2 2 2 2 2 2 2 2 2 2

Work for problem 2(a)

$$S(t) = 95\sqrt{t} \sin^2\left(\frac{1}{6}t\right) - 275 \sin^2\left(\frac{1}{3}t\right) <$$

$$S(15) = 95\sqrt{15} \sin^2\left(\frac{15}{6}\right) - 275 \sin^2\left(\frac{1}{3}(15)\right) = -121.09 \text{ gallons}$$

so, no water is ~~isn't~~ increasing but decreasing

Work for problem 2(b)

$$\int_0^{18} (95\sqrt{t} \sin^2\left(\frac{1}{6}t\right) - 275 \sin^2\left(\frac{1}{3}t\right)) dt = \boxed{109.79} \text{ gallons}$$

Continue problem 2 on page 7.

2 2 2 2 2 2 2 2 2 2

Work for problem 2(c)

$$95\sqrt{E} \sin^2\left(\frac{1}{6}t\right) - 275 \sin^2\left(\frac{1}{3}t\right) = 0$$

$$95\sqrt{E} \sin^2\left(\frac{1}{6}t\right) = 275 \sin^2\left(\frac{1}{3}t\right)$$

$$\frac{\sin^2\left(\frac{1}{6}t\right)}{\sin^2\left(\frac{1}{3}t\right)} = \frac{275}{95\sqrt{E}}$$

$$\frac{\sqrt{E} \sin^2\left(\frac{1}{6}t\right)}{\sin^2\left(\frac{1}{3}t\right)} = \frac{27}{95}$$

Work for problem 2(d)

$$\int_{18}^K (275 \sin^2\left(\frac{t}{3}\right)) dt = 0$$

GO ON TO THE NEXT PAGE.

Work for problem 3(a)

$$a) a(t) = (v(t))' = (\ln(t^2 - 3t + 3))'$$

$$a(4) = (\ln(t^2 - 3t + 3))' \Big|_{t=4} = 0,714$$

Answer: $a(4) = 0,714$

Work for problem 3(b)

3) Particle changes direction $\Leftrightarrow v(t)$ changes sign.

$$\Rightarrow v(t) = 0 \Leftrightarrow \ln(t^2 - 3t + 3) = 0;$$

Particle changes direction at $t = 1$ and $t = 2$

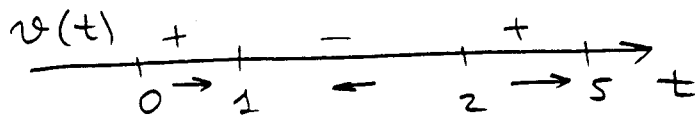
$$t^2 - 3t + 3 = 1;$$

$$t^2 - 3t + 2 = 0;$$

$$\begin{cases} t = 2 \\ t = 1 \end{cases}$$

is moving leftwards

at time $1 < t < 2$.



Continue problem 3 on page 9.

Work for problem 3(c)

$$c) \quad x(t) = \int_0^t (v(t)) dt + x(0);$$

$$x(2) = \int_0^2 (\ln(x^2 - 3x + 3)) dt + 8;$$

$$x(2) = 8,3686 \quad ;$$

Answer: $x(2) = 8,368$.

Work for problem 3(d)

$$d) \quad \text{Speed} = |v(t)|;$$

Average Speed over the interval $0 \leq t \leq 2$:

$$= \frac{1}{2} \int_0^2 (|v(t)|) dt = \frac{1}{2} \int_0^2 (|\ln(t^2 - 3t + 3)|) dt$$

$$\text{Average Speed} = 0,370509.$$

Answer: $0,371$.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

find $a(t)$
when
 $t=4$

$$0 \leq t \leq 5$$

$$v(t) = \ln(t^2 - 3t + 3) \quad x=8$$

$$t=0$$

$$a(t) = v'(t)$$

$$a(4) = v'(4)$$

$$a(4) = .714$$

Work for problem 3(b)

particle changes direction when

$v(t)$ changes from +ve \rightarrow -ve or
-ve \rightarrow +ve



-8

left + increment when

$$s(t) > 0 \text{ and } v(t) < 0$$

$$s(t) < 0 \text{ and } v(t) < 0$$

so when $v(t) = 0$

$$\ln(t^2 - 3t + 3) = 0$$

$$t = 1, 2$$

The particle changes

direction when $t = 1, 2$.

Continue problem 3 on page 9.

Work for problem 3(c)

$$t=2$$

$$S(t) = ?$$

$$\int v(t) dt = S(t)$$

$$S(2) = \int_0^2 \ln(t^2 - 3t + 3) dt$$

$$S(2) = .369$$

$$\text{at } t=0, x=8$$

$$\text{So } .369 + 8 = \underline{\underline{8.369}}$$

Work for problem 3(d)

$$\text{Av speed} = \frac{1}{2-0} \int_0^2 \ln(t^2 - 3t + 3) dt$$

$$= .184$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Work for problem 3(a)

$$v(t) = \ln(t^2 - 3t + 3), \quad 0 \leq t \leq 5$$

$$a(t) = v'(t) = \frac{1}{t^2 - 3t + 3} \cdot (2t - 3) = \frac{2t - 3}{t^2 - 3t + 3}$$

$$a(4) = \frac{5}{16 - 12 + 3} = \frac{5}{7}$$

Work for problem 3(b)

a particle changes direction when

$$v(t) = \ln(t^2 - 3t + 3) = 0$$

using calculator: $t = 2, t = 1$

in times $t=1, t=2$ particle changes its direction.

Particle moves to the left in the interval between $t=1$ and $t=2$,
i.e. $t \in (1, 2)$

Work for problem 3(c)

$s(t)$ - the position of particle .

$$s(t) = \int v(t) dt = \int \ln(t^2 - 3t + 3) dt = \frac{-3 \ln |t^2 - 3t + 3|}{2} + t \ln(t^2 - 3t + 3)$$

$$2t + \frac{\sqrt{3} \pi \cdot \tan^{-1} \left(\frac{\sqrt{3}(2t-3)}{3} \right)}{180}$$

$$s(2) = \frac{-3 \ln |1|}{2} + 2 \ln 1 - 4 + \frac{\sqrt{3} \pi \tan^{-1} \left(\frac{\sqrt{3}}{3} \right)}{180} = \frac{\sqrt{3} \cdot \pi \cdot 30}{180} - 4 = \frac{\sqrt{3} \pi}{6} - 4 = -3.09$$

$$s(0) = \frac{-3 \ln 3}{2} + \frac{\sqrt{3} \pi \cdot -60}{180} = -\frac{3}{2} \ln 3 + \frac{\sqrt{3}}{2} \pi$$

Work for problem 3(d)

speed = |velocity|

$$\text{Average speed} = \left| \frac{s(2) - s(0)}{2-0} \right| = \left| \frac{-3.09 - s(0)}{2} \right| = \left| \frac{-3.09 + 1.34}{2} \right| = 0.86$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

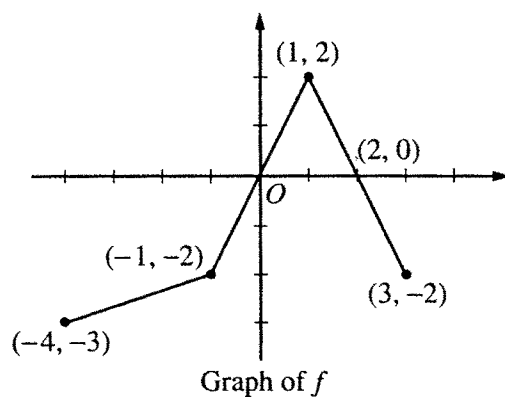
NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$g(-1) = \int_{-4}^{-1} f(t) dt$$

$$= \frac{-3 + (-2)}{2} (3)$$

$$= \frac{-15}{2}$$

$$g'(-1) = f(-1)$$

$$= -2$$

$$g''(-1) = f'(-1)$$

$g''(-1)$ does not exist.

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$g(x)$ has an inflection pt when
 $x=1$

$$g''(x) = f'(x)$$

inflection pt occurs when $f'(x)$ goes from + to - or from - to +
 (concavity change). This happens when $x=1$

Work for problem 4(c)

$$h(x) = \int_x^3 f(t) dt$$

$h(x)$ is 0 when $x=3, 1, -1$

$\int_3^3 f(t) dt, \int_1^3 f(t) dt, \int_{-1}^3 f(t) dt$ are all zero.

Work for problem 4(d)

$h(x)$ decreases when $0 < x < 2$.

$$h'(x) = -f(x) \quad (h(x) = \int_x^3 f(t) dt)$$

$\therefore h'(x) < 0$ when $f(x) > 0$.

$f(x) > 0$ when $0 < x < 2$.

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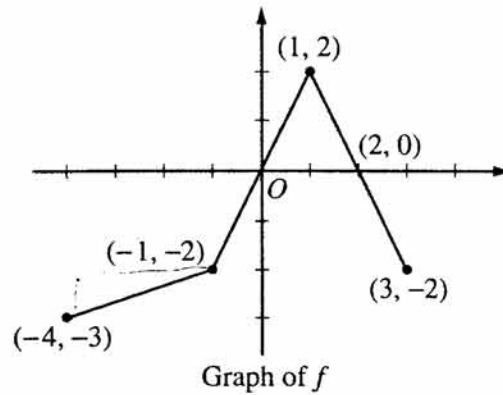
NO CALCULATOR ALLOWED

CALCULUS AB
SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$g(-1) = \int_{-4}^{-1} f(t) dt = -\frac{3}{2} - 6 = -7.5$$

$$g'(-1) = f(-1) = -2$$

$g''(-1)$ does not exist, since $f'(-1)$ does not exist
 ($f(x)$ is not differentiable at point $x = -1$)

Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$g''(x) = f'(x)$$

$g''(x) = 0 \Rightarrow f'(x) = 0$, but there are no points where $f'(x) = 0$ on the interval $(-4; 3) \Rightarrow$ there are no points of inflection of function $g(x)$ on the same interval $(-4; 3)$

Work for problem 4(c)

there is only ~~one~~ ^{two} values of $x \Rightarrow$

$$x = 1, x = -1$$

$$h(x) = \int_x^3 f(t) dt$$

$$h(1) = 1 - 1 = 0, h(-1) = 1 - 1 + 1 - 1 = 0$$

Work for problem 4(d)

$$h'(x) = \left(- \int_3^x f(t) dt \right)' = -f(x)$$

$h'(x) < 0$ for $h(x)$ to decrease $\Rightarrow f(x) > 0 \Rightarrow x \in [0; 2]$

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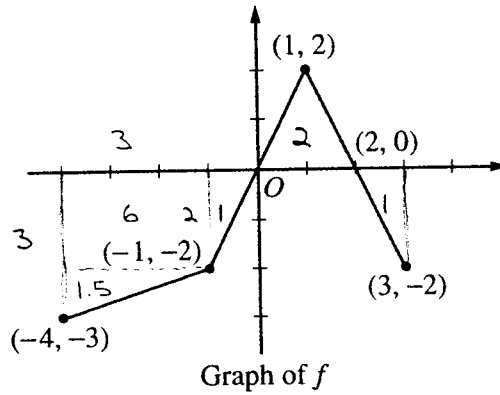
NO CALCULATOR ALLOWED

**CALCULUS AB
SECTION II, Part B**

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



$g'(x) = f(x)$
 $g''(x) = f'(x)$

Work for problem 4(a)

$$g(-1) = \int_{-4}^{-1} f(t) dt = 7.5$$

$$g'(-1) = -2$$

$$g''(-1) = \text{does not exist}$$

Continue problem 4 on page 11.

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C₂

NO CALCULATOR ALLOWED

Work for problem 4(b)

$g(x)$ experiences a point of inflection where $g''(x) = 0$ ($f'(t) = 0$), hence it is where $f(t)$ has critical points. $x = 1$

Work for problem 4(c)

$x = -1$, where $h(x) = 0$

Work for problem 4(d)

where $h'(x) = \text{negative}$, hence $(-4, -1)(-1, 0)(2, 3)$.

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 5(a)

$$2y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$(2y - x) \frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

Work for problem 5(b)

 $\frac{dy}{dx} = \text{slope of the tangent}$

$$\frac{1}{2} = \frac{y}{2y - x}$$

$$2y = 2y - x \Rightarrow x = 0$$

$$x = 0 \Rightarrow y^2 = 2 \Rightarrow y = +\sqrt{2} \text{ or } -\sqrt{2}$$

$(0, \sqrt{2})$ and $(0, -\sqrt{2})$ are points on the curve where the tangent has slope $\frac{1}{2}$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

line tangent is horizontal \Rightarrow slope of tangent is zero $\Rightarrow \frac{dy}{dx} = 0$

if ~~for~~ $\frac{dy}{dx} = 0$, then $y = 0$

substituting $y = 0$ in $y^2 = 2 + xy$ gives $0 = 2$ which is false

$\therefore \frac{dy}{dx}$ cannot be zero \Rightarrow there is no point (x, y) where the line tangent to the curve is horizontal

Work for problem 5(d)

$$2y \frac{dy}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x$$

$$y = 3 \Rightarrow 9 = 2 + 3x \Rightarrow 3x = 7 \Rightarrow x = \frac{7}{3}$$

at $t = 5$:

$$2(3)(6) = \frac{dx}{dt} (3) + (6) \left(\frac{7}{3}\right)$$

$$36 = 3 \frac{dx}{dt} + 14$$

$$\frac{dx}{dt} = \frac{36 - 14}{3}$$

$$\frac{dx}{dt} = \frac{22}{3}$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 5(a)

$$y^2 = 2 + xy$$

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

$$2y \frac{dy}{dx} = \frac{dx}{dx} y + x \frac{dy}{dx}$$

$$2y \frac{dy}{dx} = y + x \frac{dy}{dx}$$

$$2y \frac{dy}{dx} - x \frac{dy}{dx} = y$$

$$\frac{dy}{dx} (2y - x) = y$$

$$\boxed{\frac{dy}{dx} = \frac{y}{2y-x}}$$

Work for problem 5(b)

$$(x_1, y_1) \quad y - y_1 = m(x - x_1)$$

$$\frac{y}{2y-x} = \frac{1}{2}$$

$$2y = 2y - x$$

$$y = y - \frac{1}{2}x$$

$$y + \frac{1}{2}x = y$$

$$\frac{1}{2}x = 0$$

$$x = 0$$

$$y^2 = 2 + 0(4)$$

$$y^2 = 2 + 0$$

$$y = \pm\sqrt{2}$$

$$y^2 = 2 + xy$$

$$\boxed{\begin{matrix} (0, \sqrt{2}) \\ (0, -\sqrt{2}) \end{matrix}}$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$f'(c) = 0$$

$$\frac{y}{2y-x} = 0$$

$$y = 0(2y-x)$$

$$\frac{y}{2y-x} = 0 \quad \frac{0}{-x} = 0$$

$y = 0$ can't solve for
 x at $f'(c) = 0$.

Work for problem 5(d)

$$y^2 = 2 + xy$$

$$2y \frac{dy}{dt} = \frac{dx}{dt} y + \frac{dy}{dt} x$$

$$y = 3$$

$$\frac{dy}{dt} = 6$$

$$t = 5$$

$$\frac{2y \frac{dy}{dt} - \frac{dy}{dt} x}{y} = \frac{dx}{dt}$$

$$\frac{2(3)(6) - 6(5)}{3} = \frac{dx}{dt}$$

$$\frac{36 - 30}{3} = \frac{dx}{dt}$$

$$\frac{6}{3} = \frac{dx}{dt} = 2$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 5(a)

$$y^2 = 2 + xy$$

$$2y \frac{dy}{dx} = x \frac{dy}{dx} + y$$

$$(2y - x) \frac{dy}{dx} = y$$

$$\frac{dy}{dx} = \frac{y}{2y - x}$$

Work for problem 5(b)

$$\frac{1}{2} = \frac{y}{2y - x}$$

$$-x = 2y$$

$$x = 0$$

$$+ (x = 0, y = \sqrt{2})$$

$$(0, \sqrt{2})$$

Continue problem 5 on page 13.

NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{y}{2y-x}$$

$$y^2 = 2 + xy$$

$$\therefore 2y \neq x$$

Work for problem 5(d)

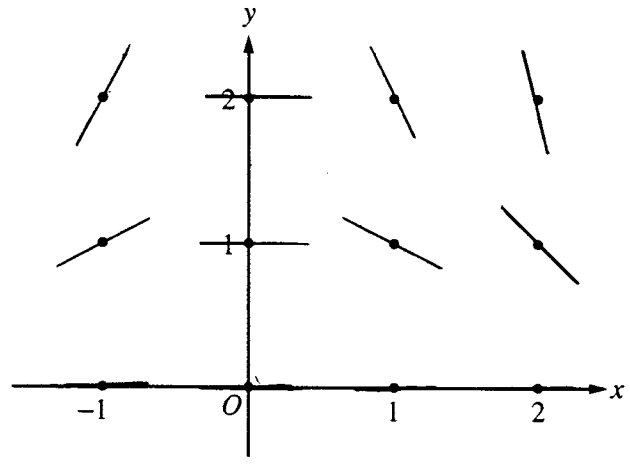
$$y^2 = 2 + xy$$

$$2y \frac{dy}{dt} = x \frac{dy}{dt} + y \frac{dx}{dt}$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

Work for problem 6(a)



$$\frac{dy}{dx} = \frac{-xy^2}{2}$$

$$\frac{-(-2)(1)}{2}$$

$$\frac{-(-2)(4)}{2}$$

$$\frac{-(-1)(1)}{2} \quad \frac{-(-1)(4)}{2}$$

Work for problem 6(b)

$$f(-1) = 2$$

$$\frac{dy}{dx} = \frac{-xy^2}{2} = \frac{-(-1)(4)}{2} = 2$$

$$y = m(x - x_1) + y_1$$

$$y = 2(x - (-1)) + 2$$

$$\underline{\underline{y = 2x + 4}}$$

Continue problem 6 on page 15.

Work for problem 6(c)

$$\frac{dy}{dx} = \frac{-xy^2}{2}$$

$$2dy = -xy^2 dx$$

$$\frac{2}{y^2} dy = x dx$$

$$-2(y^{-2}) dy = \int x dx$$

$$2y^{-1} = \frac{1}{2}x^2 + C$$

$$-1 = 2 \quad \Bigg\}$$

$$2(2^{-1}) = \frac{1}{2}(-1)^2 + C$$

$$1 = \frac{1}{2} + C$$

$$C = \frac{1}{2}$$

$$2y^{-1} = \frac{1}{2}x^2 + \frac{1}{2}$$

$$\frac{1}{y} = \frac{1}{4}x^2 + \frac{1}{4}$$

$$\frac{1}{y} = \frac{x^2 + 1}{4}$$

$$y = \frac{4}{x^2 + 1}$$

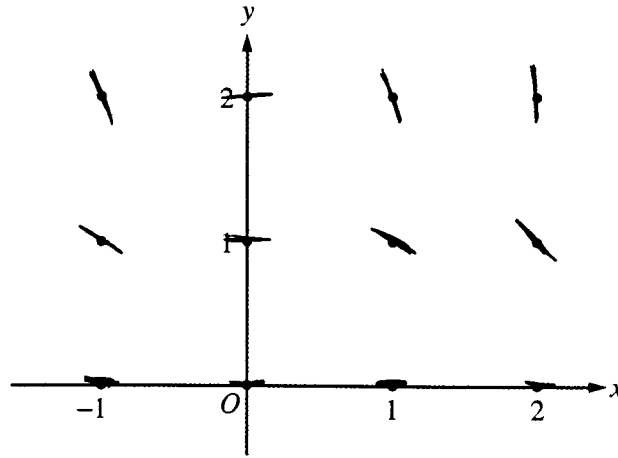
END OF EXAM

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NO CALCULATOR ALLOWED

Work for problem 6(a)



dy/dx	x	y
0	0	0
$-\frac{1}{2}$	1	1
-2	1	2
-1	2	1
-4	2	2
$\frac{1}{2}$	-1	1
2	-1	2

Work for problem 6(b)

$$y - 2 = m(x + 1)$$

$$+ \frac{4}{2} = 2$$

$$y - 2 = 2(x + 1)$$

$$\boxed{y = 2x}$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\int \frac{dy}{y^2} = \int \frac{-x dx}{2}$$

$$\ln|y^2| = -\frac{1}{2}x^2 + C$$

~~Handwritten scribbles~~

$$\ln 4 = -\frac{1}{2} + C$$

$$4 = ce^{-1/2}$$

$$4e^{1/2} = C$$

$$y^2 = ce^{-x^2/4}$$

$$y = \sqrt{ce^{-x^2/4}}$$

$$y = \sqrt{4e^{1/2} e^{-x^2/4}}$$

~~Handwritten scribbles~~

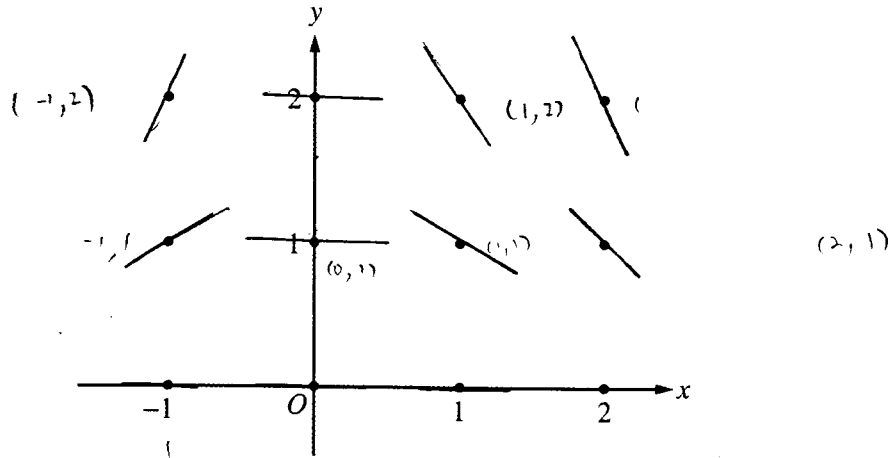
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NO CALCULATOR ALLOWED

Work for problem 6(a)



Work for problem 6(b)

$(-1, 2)$

$$\frac{dy}{dx} = \frac{1 \times 4}{2} = 2$$

$$y = 2(x+1) + 2$$

Continue problem 6 on page 15.

NO CALCULATOR ALLOWED

Work for problem 6(c)

$$\frac{dy}{dx} = \frac{-xy^2}{2}$$

$$2dy = -xy^2 dx$$

$$2y = -\frac{1}{3x} y^3$$

$$= -\frac{1}{3} xy^3 + C$$

$$(-1, 2)$$

$$4 = \frac{8}{3} + C$$

$$C = \frac{12}{3} - \frac{8}{3}$$

$$= \frac{4}{3}$$

$$2y = -\frac{1}{3} xy^3 + \frac{4}{3}$$

$$y = -\frac{1}{6} xy^3 + \frac{4}{6}$$

END OF EXAM

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