

AP[®] STATISTICS

2012 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) compare two distributions presented with histograms; (2) comment on the appropriateness of using a two-sample t -procedure in a given setting.

Solution

Part (a):

Household size tended to be larger in 1950 than in 2000. The histograms reveal a much larger proportion of small (1-, 2-, and 3-person) households in 2000 than in 1950. Similarly, the histograms reveal a much smaller proportion of large (5-person and larger) households in 2000 than in 1950. Also, the median household sizes can be calculated to be 5 people per household in 1950 compared with 3 or 4 people per household in 2000. The year 1950 displayed slightly more variability in household sizes than the year 2000. Although the interquartile ranges for both years are the same (3 people), the standard deviation (1950: about 2.6 people; 2000: about 2.1 people) and the range (1950: 13 people; 2000: 11 people) are larger for 1950 than for 2000. Both distributions of household size are skewed to the right. In both years, there are a few households with very large families, as large as 14 people in 1950 and 12 people in 2000.

Part (b):

The conditions for applying a two-sample t -procedure are:

1. The data come from independent random samples or from random assignment to two groups;
2. The populations are normally distributed, or both sample sizes are large;
3. The population sizes are at least 10 (or 20) times the sample sizes.

The first condition is satisfied because independent random samples were selected for the years 1950 and 2000. The second condition is satisfied because the sample sizes (500 in each group) are quite large, despite the right skewness of the distributions of household sizes in the sample data. The third condition is satisfied because the number of households in the large metropolitan area in both 1950 and 2000 would easily exceed $10 \times 500 = 5,000$.

Scoring

This question is scored in four sections. Part (a) has three components: (1) comparing the centers of the two distributions; (2) comparing variability for the two distributions; (3) identifying the shapes of both distributions and including context related to the variable of interest. Section 1 consists of part (a), component 1; section 2 consists of part (a), component 2; section 3 consists of part (a), component 3. Section 4 consists of part (b). Sections 1 and 2 are scored as essentially correct (E) or incorrect (I). Sections 3 and 4 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if the response correctly compares center (or location) for both distributions.

Incorrect (I) otherwise.

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Question 3 (continued)

Section 2 is scored as follows:

Essentially correct (E) if the response correctly compares variability for both distributions.

Incorrect (I) otherwise.

Section 3 is scored as follows:

Essentially correct (E) if the response includes context related to the variable of interest (household size) *AND* the response correctly identifies the shapes of both distributions.

Partially correct (P) if the response correctly identifies the shapes of both distributions *BUT* does *NOT* include context related to the variable of interest (household size),

OR

if the response correctly identifies the shape of only one distribution *AND* includes context related to the variable of interest (household size).

Incorrect (I) otherwise.

Section 4 is scored as follows:

Essentially correct (E) if the response correctly states and checks the following two conditions.

1. The data come from independent random samples
2. Normality/sample size conditions.

Partially correct (P) if the response correctly states and checks only one of the two conditions listed above,

OR

if the response correctly refers to random samples and large sample size, *BUT* does *NOT* state and check either condition correctly.

Incorrect (I) otherwise.

Note: The population size condition does not need to be checked to earn E or P.

Each essentially correct (E) section counts as 1 point. Each partially correct (P) section counts as $\frac{1}{2}$ point.

| | |
|----------|-----------------------------|
| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |
| 1 | Minimal Response |

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication.

(a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000.

The shape of the household size distribution for 1950 is slightly skewed right, whereas the distribution for 2000 is strongly skewed right. The center of the 1950 distribution is higher than that of the 2000 distribution (≈ 5 compared to ≈ 3). Lastly, the spread of the 1950 distribution is also larger than that of 2000. The spread of 1950 is from 1- ≈ 14 , whereas 2000 is only from 1- ≈ 12 .

(b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using a two-sample *t*-procedure, and explain whether the conditions for inference are met.

The conditions are as follows:

- 1) Two independent simple random samples
 - yes, this condition is met (stated in problem that the samples are independent and random)
- 2) large sample size OR normal dist.
 $n_1 \geq 30$ $n_2 \geq 30$
 $500 \geq 30 \checkmark$ $500 \geq 30 \checkmark$
 - yes, sample size is large enough.

Even though the distributions aren't normal, the sample sizes are large enough to complete the inference.

(a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000.

shape - Both distributions are skewed right; distribution of Year 2000 has a stronger skew

Center - The center has moved slightly ^(left) lower from 1950's to 2000's distribution.

spread - the spread of 1950's distribution is slightly greater than that of 2000's.

(b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using a two-sample *t*-procedure, and explain whether the conditions for inference are met.

Pre conditions:

- (✓) 1. $n < 10\% N$ → assumed that there are more than 5000 households in the United States.
- (X) 2. Approximately normally distributed → these distributions are significantly skewed. condition is not met.
- 3. Random sample → given in the question ("Independent random samples of 500 households...")

- (a) Compare the distributions of household size in the metropolitan area for the years 1950 and 2000.

The distribution of household size in 1950 is slightly skewed to the right, but does not show any extreme outliers. It also includes households with sizes between 1 and 14.

The distribution of household size in 2000 is extremely skewed to the right and households with 10-12 people may be considered outliers. This distribution includes households with sizes between 1 and 12, which is different than the 1950's distribution.

- (b) A researcher wants to use these data to construct a confidence interval to estimate the change in mean household size in the metropolitan area from the year 1950 to the year 2000. State the conditions for using a two-sample t -procedure, and explain whether the conditions for inference are met.

The conditions for a 2-sample t -procedure are as follows:

- * The samples of households must be random.
- This is stated in the problem.
- * The samples must also be independent.
- This is also stated in the problem.
- * The sample size must be greater than or equal to 40. In both household samples, the number of samples are both 500, which is greater than or equal to 40.

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2012 SCORING COMMENTARY

Question 3

Overview

The primary goals of this question were to assess students' ability to (1) compare two distributions presented with histograms; (2) comment on the appropriateness of using a two-sample t -procedure in a given setting.

Sample: 3A

Score: 4

In part (a) the student effectively compares the center and variability of the two distributions, with appropriate numerical support. As a result, sections 1 and 2 were scored as essentially correct. The student also provides a thorough, accurate description of the shapes of the two distributions, with clear reference to household size. Consequently, section 3 was scored as essentially correct. In part (b) the student eloquently states and correctly verifies the random samples and normality/large sample size conditions, and section 4 was scored as essentially correct. With all four sections scored as essentially correct, the response earned a score of 4.

Sample: 3B

Score: 3

In part (a) the student begins by correctly identifying the shapes of the two distributions. However, the student never mentions the variable of interest (household size). As a result, section 3 was scored as partially correct. The student provides an accurate comparison of both center and variability based on the two histograms, so sections 1 and 2 were each scored as essentially correct. In part (b) the student correctly states and checks the random samples condition. However, the student does not address the normality/large sample size condition correctly, suggesting that the condition is not met in spite of the large sample sizes. Consequently, section 4 was scored as partially correct. With two sections scored as essentially correct and two sections scored as partially correct, the response earned a score of 3.

Sample: 3C

Score: 2

In part (a) the student separately describes the household size distributions in 1950 and 2000 but never compares their centers and spreads. As a result, sections 1 and 2 were each scored as incorrect. However, the student does correctly describe the shapes of the distributions and includes the context (household size), so section 3 was scored as essentially correct. In part (b) the student accurately states and checks both the independent random samples and normality/large sample size conditions. Consequently, section 4 was scored as essentially correct. With two sections scored as essentially correct and two sections scored as incorrect, the response earned a score of 2.