Question 2

Intent of Question

The primary goals of this question were to assess students' ability to (1) perform calculations and compute expected values related to a discrete probability distribution; (2) implement a normal approximation based on the central limit theorem.

Solution

Part (a):

By counting the number of sectors for each value and dividing by 10, the probability distribution is calculated to be:

x	\$2	\$1	–\$8
$P(x)$	0.6	0.3	0.1

Part (b):

The expected value of the net contribution for one play of the game is:

$E(x) = \$2(0.6) + \$1(0.3) + (-\$8)(0.1) = \$0.70$ (or 70 cents).

Part (c):

The expected contribution after n plays is $\$0.70n$. Setting this to be at least \$500 and solving for n gives:

$0.70n \geq 500$, so $n \geq \frac{500}{0.70} \approx 714.286$,

so 715 plays are needed for the expected contribution to be at least \$500.

Part (d):

The normal approximation is appropriate because the very large sample size $(n = 1{,}000)$ ensures that the central limit theorem holds. Therefore, the sample mean of the contributions from 1,000 plays has an approximately normal distribution, and so the sum of the contributions from 1,000 plays also has an approximately normal distribution.

The z-score is $\frac{500 - 700}{92.79} \approx -2.155$.

The probability that a standard normal random variable exceeds this z-score of -2.155 is 0.9844. Therefore, the charity can be very confident about gaining a net contribution of at least \$500 from 1,000 plays of the game.

Question 2 (continued)

Scoring

This question is scored in three sections. Section 1 consists of parts (a) and (b); section 2 consists of part (c); and section 3 consists of part (d). Sections 1, 2, and 3 are scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1 is scored as follows:

Essentially correct (E) if all three probabilities are filled in correctly in the table in part (a) *AND* the expected value is calculated correctly in part (b), with work shown.

Partially correct (P) if all three probabilities are filled in correctly in the table in part (a) *AND* the expected value is not calculated correctly in part (b),
OR
the probabilities in part (a) are not all correct *AND* the expected value in part (b) is calculated appropriately from the probabilities given in part (a) or from the correct probabilities.

Incorrect (I) if the response does not meet the criteria for E or P.

Section 2 is scored as follows:

Essentially correct (E) if the response addresses the following two components:
1. Provides a solution based on a reasonable calculation, equation, or inequality from the answer given in part (b).
2. Clearly selects the next higher integer as the answer.

Partially correct (P) if the response correctly completes component (1) listed above but not component (2).

Incorrect (I) if the response does not meet the criteria for E or P.

Section 3 is scored as follows:

Essentially correct (E) if the response correctly addresses the following three components:
1. Indicates the use of a normal distribution with the correct mean and standard deviation.
2. Uses the correct boundary and indicates the correct direction.
3. Has the correct normal probability consistent with components (1) and (2).

Partially correct (P) if the response correctly addresses exactly two of the three components listed above.

Incorrect (I) if the response does not meet the criteria for E or P.

Question 2 (continued)

Notes

- Because the question asks students to use a normal distribution and specifies the parameter values, the response does not have to justify the normal approximation or show how to calculate the parameter values.
- If the response earns credit for component (1) but no direction has been provided for component (2), then the response earns credit for component (3) if the correct probability of 0.9844 is reported.
- If the response does not earn credit for component (1) owing to incorrect identification of the mean and/or standard deviation, then the response can still earn credit for component (2) if the boundary is calculated correctly from the mean and standard deviation indicated in component (1).

4 Complete Response

All three sections essentially correct

3 Substantial Response

Two sections essentially correct and one section partially correct

2 Developing Response

Two sections essentially correct and one section incorrect

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

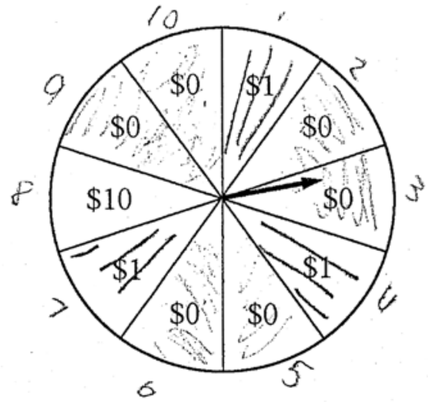
1 Minimal Response

One section essentially correct and two sections incorrect

OR

Two sections partially correct and one section incorrect

2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	−$8
$P(x)$	.6	.3	.1

(b) What is the expected value of the net contribution to the charity for one play of the game?

$\sum(X) = (\$2 \times .6) + (\$1 \times .3) + (-\$8 \times .1)$

$\sum(X) = \$1.2 + \$.3 + -\$.8$

$\sum(X) = \$.7$

Let X represent the net contribute to the charity when one person plays the game once.

The expected value of X is $0.7 to the charity for one play of the game.

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(c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?

$X = \$500$

$n = ?$

$\$0.7 \times n \geq \$500$

$n \geq 714.286$

The fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 is 715 games

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

$P(X \geq \$500)$

$\sigma = \$92.79$

$z = \dfrac{\$500 - \$700}{92.79} = -2.155$

$P(z > -2.155) = .984$

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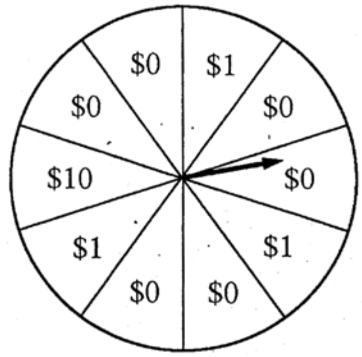
2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	–$8
$P(x)$	.6	.3	.1

(b) What is the expected value of the net contribution to the charity for one play of the game?

E(net) = (.6)(2) + (.3)(1) − (.1)(8) = $0.70

The expected value of the net contribution to the charity for one play of the game is $0.70.

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(c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?

Cont \$500

each game 0.70

$\# \text{ games} = \frac{\text{total cont}}{\text{each game}}$

$\# \text{ games} = \frac{500}{0.70}$

\# games = 714.2857

The fewest number of games played would be 715 times.

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

$N(\$700, 92.79)$

$t_{999} = \frac{\bar{x} - \mu}{SD} = \frac{500 - 700}{\$92.79} = -2.1554$

$P(t_{999} > -2.1554) = .9844$

The approximate probability that the charity would obtain a net contributiof of at least \$500 in 1000 plays of the game is .98443.

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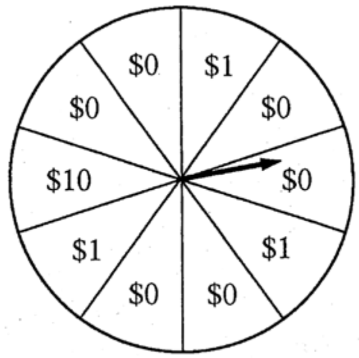
2. A charity fundraiser has a Spin the Pointer game that uses a spinner like the one illustrated in the figure below.



A donation of $2 is required to play the game. For each $2 donation, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 10 sectors.

(a) Let X represent the net contribution to the charity when one person plays the game once. Complete the table for the probability distribution of X.

x	$2	$1	–$8
$P(x)$	6/10 = 60%	3/10 = 30%	1/10 = 10%

(b) What is the expected value of the net contribution to the charity for one play of the game?

$(.6 \cdot 2) + (.3 \cdot 1) + (.1 \cdot -8) = \$.70$

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(c) The charity would like to receive a net contribution of \$500 from this game. What is the fewest number of times the game must be played for the expected value of the net contribution to be at least \$500 ?

$500/.7 = 714.29$

The game must be played 715 times at least

(d) Based on last year's event, the charity anticipates that the Spin the Pointer game will be played 1,000 times. The charity would like to know the probability of obtaining a net contribution of at least \$500 in 1,000 plays of the game. The mean and standard deviation of the net contribution to the charity in 1,000 plays of the game are \$700 and \$92.79, respectively. Use the normal distribution to approximate the probability that the charity would obtain a net contribution of at least \$500 in 1,000 plays of the game.

normalcdf(500, 1E99, 700, 92.79) = .984

or 98.4%

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Question 2

Overview

The primary goals of this question were to assess students' ability to (1) perform calculations and compute expected values related to a discrete probability distribution; (2) implement a normal approximation based on the central limit theorem.

Sample: 2A
Score: 4

In part (a) the student correctly identifies all three probabilities and fills in the table for the probability distribution of X correctly. In part (b) the student calculates the correct expected value, with work clearly shown. The student indicates that $0.70 is the expected value of the net contribution to the charity for one play of the game. Parts (a) and (b) together were scored as essentially correct for section 1. In part (c) the student sets up the appropriate inequality and solves for n, satisfying one component of section 2. The student finds that the game must be played at least 714.286 times for the expected value of the net contribution to be at least $500. The student then selects the next higher integer, 715, as the fewest number of times the game must be played, and the second component is satisfied. Both components of section 2 are satisfied, and section 2 was scored as essentially correct. In part (d) the student clearly indicates that the normal distribution with the correct mean and standard deviation is used by providing a correct z-score calculation and by providing a sketch of a normal distribution with the mean and standard deviation labeled. Thus, one component of section 3 is satisfied in two ways. The correct boundary and direction are given by each of the two probability statements, satisfying a second component. The correct normal probability calculation of 0.984 is reported, satisfying the third component. All three components of section 3 are satisfied, and section 3 was scored as essentially correct. Because all three sections were scored as essentially correct, the response earned a score of 4.

Sample: 2B
Score: 3

In part (a) the student correctly identifies all three probabilities and fills in the table for the probability distribution of X correctly. In part (b) the student calculates the correct expected value, with work clearly shown. The student indicates that $0.70 is the expected value of the net contribution to the charity for one play of the game. Parts (a) and (b) together were scored as essentially correct for section 1. In part (c) the student provides a solution based on a reasonable calculation from the answer reported in part (b), thus satisfying one component of section 2. The student then selects the next higher integer as the answer and satisfies the second component. The student also clearly communicates that the fewest number of times the game must be played is 715. Both components of section 2 are satisfied, and section 2 was scored as essentially correct. In part (d) the student clearly indicates the use of the normal distribution with the mean and standard deviation identified by using the standard notation, $N(\$700, 92.79)$. However, the student then proceeds to use the t-distribution with 999 degrees of freedom. Although the t-distribution with a large number of degrees of freedom is a good approximation to the normal distribution, the stem of the problem directs students to use the normal distribution. Use of the t-distribution is not correct, and the response does not satisfy the condition for one component of section 3. The student indicates the correct direction by using the inequality and the correct boundary of -2.155, thus satisfying a second component. The correct normal probability calculation of 0.9844 is reported, satisfying the third component. Two of the three components of section 3 are satisfied, and section 3 was scored as partially correct. Because two sections were scored as essentially correct and one section was scored as partially correct, the response earned a score of 3.

Question 2 (continued)

Sample: 2C
Score: 2

In part (a) the student correctly identifies the three probabilities and fills in the table for the probability distribution of X correctly. In part (b) the student calculates the correct expected value, with work clearly shown. Parts (a) and (b) together were scored as essentially correct for section 1. In part (c) the student provides a solution based on a reasonable calculation from the answer reported in part (b), satisfying one component of section 2. The student then selects the next higher integer as the answer, satisfying the second component. The student also clearly communicates that the fewest number of times the game must be played for the expected value of the net contribution to be at least $500 is 715. Both components of section 2 are satisfied, and section 2 was scored as essentially correct. In part (d) the student uses calculator notation to answer the question. The student indicates use of a normal distribution but does not clearly identify the mean and standard deviation. Calculator notation in itself does not identify parameters, so the response does not satisfy this component of section 3. The student does not identify the bound or the direction. Unless the student identifies the lower bound and the upper bound of the interval, the calculator notation itself is not sufficient, and another component is not satisfied. The student reports the correct normal probability of 0.984, satisfying the third component. Only one of the three components of section 3 is satisfied, so section 3 was scored as incorrect. Because two sections were scored as essentially correct and one section was scored as incorrect, the response earned a score of a 2.