

AP[®] PHYSICS

2012 SCORING GUIDELINES

General Notes About 2012 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.
2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded in part (b). One exception to this practice may occur in cases where the numerical answer to a later part should easily be recognized as wrong, for example, a speed faster than the speed of light in vacuum.
3. Implicit statements of concepts normally receive credit. For example, if the use of an equation expressing a particular concept is worth 1 point, and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exam equation sheets. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics Course Description*.
4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.
5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer owing to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will eliminate the level of accuracy required to determine the difference in the numbers, and some credit may be lost.

**AP[®] PHYSICS C: MECHANICS
2012 SCORING GUIDELINES**

Question 3

15 points total

**Distribution
of points**

(a)

i. 3 points

For starting with Newton's second law for translation, with friction as the net force

$$\Sigma F = -f = Ma$$

1 point

For a correct expression for the frictional force

$$f = \mu Mg$$

1 point

For indicating that linear acceleration is the time derivative of velocity

$$a = \frac{dv}{dt}$$

1 point

$$\frac{dv}{dt} = -\mu g$$

ii. 3 points

For starting with Newton's second law for rotation, with a correct substitution for the rotational inertia

$$\tau = MR^2\alpha$$

1 point

For a correct expression for the torque, using the frictional force

$$\tau = \mu MgR$$

1 point

For indicating that the angular acceleration is the time derivative of the angular velocity

$$\alpha = \frac{d\omega}{dt}$$

1 point

$$\frac{d\omega}{dt} = \frac{\mu g}{R}$$

(b)

i. 2 points

For setting up the integral of the function determined in part (a)-i

$$\int_{v_0}^v dv = -\int_0^t \mu g dt$$

1 point

For the correct answer

$$v = v_0 - \mu gt$$

1 point

Alternate solution

Alternate points

For a clear substitution of the acceleration from part (a)-i into the kinematics equation

1 point

$$a = -\mu g$$

$$v = v_0 + at$$

For the correct answer

1 point

$$v = v_0 - \mu gt$$

**AP[®] PHYSICS C: MECHANICS
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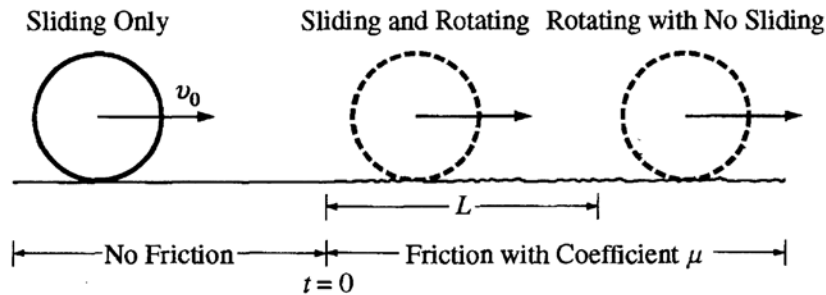
Question 3 (continued)

	Distribution of points
(b) continued	
ii. 2 points	
For setting up the integral of the function determined in part (a)-ii	1 point
$\int_0^{\omega} d\omega = \int_0^t (\mu g/R) dt$	
For the correct answer	1 point
$\omega = \mu g t / R$	
<i>Alternate solution</i>	<i>Alternate points</i>
<i>For a clear substitution of the angular acceleration from part (a)-ii into the correct rotational kinematics equation</i>	<i>1 point</i>
$\alpha = \frac{\mu g}{R}$	
$\omega = \omega_0 + \alpha t$	
<i>For the correct answer</i>	<i>1 point</i>
$\omega = \mu g t / R$	
(c) 2 points	
For indicating that the linear speed is equal to $R\omega$ when the slipping stops	1 point
$v = R\omega$	
$v_0 - \mu g t = R \left(\frac{\mu g t}{R} \right)$	
For the correct answer	1 point
$t = \frac{v_0}{2\mu g}$	
(d) 1 point	
For substituting the time found in part (c) into a correct kinematics equation	1 point
$v = v_0 - \mu g \left(\frac{v_0}{2\mu g} \right)$	
$v = v_0 / 2$	

**AP[®] PHYSICS C: MECHANICS
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Question 3 (continued)

		Distribution of points
(e)	2 points	
	For setting up the integral of the velocity function determined in part (b)-i	1 point
	$L = \int_0^t (v_0 - vgt) dt$	
	For the correct answer, with correct supporting work	1 point
	$L = \left[v_0 t - \frac{1}{2} \mu g t^2 \right]_0^{\frac{v_0}{2\mu g}}$	
	$L = \frac{3v_0^2}{8\mu g}$	
	<i>Alternate solution #1</i>	<i>Alternate points</i>
	For substituting the velocity from part (d) and the acceleration from part (a)-i into a correct equation that solves for L	1 point
	$v^2 = v_0^2 + 2a\Delta x$	
	$\left(\frac{v_0}{2} \right)^2 = v_0^2 + 2(-\mu g)L$	
	For the correct answer, with correct supporting work	1 point
	$L = \frac{3v_0^2}{8\mu g}$	
	<i>Alternate solution #2</i>	<i>Alternate points</i>
	For substituting the velocity from part (d) and the acceleration from part (a)-i into a correct equation that solves for L	1 point
	<u>Note:</u> The time determined in part (c) must also be substituted.	
	$\Delta x = v_0 t + \frac{1}{2} at^2$	
	$L = v_0 \left(\frac{v_0}{2\mu g} \right) + \frac{1}{2} (-\mu g) \left(\frac{v_0}{2\mu g} \right)^2$	
	For the correct answer, with correct supporting work	1 point
	$L = \frac{3v_0^2}{8\mu g}$	



Mech. 3.

A ring of mass M , radius R , and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L , the ring begins rolling without sliding. Express all answers to the following in terms of M , R , v_0 , μ , and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$ma = -\mu mg$$

$$\frac{dv}{dt} = -\mu g$$

ii. The angular velocity ω of the ring as a function of time t

$$\tau = Fr = I\alpha$$

$$\mu mgr = I \frac{d\omega}{dt}$$

$$\frac{d\omega}{dt} = \frac{\mu mgr}{MR^2} = \frac{\mu g}{R}$$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$\frac{dv}{dt} = -\mu g \quad \int_{v_0}^v dv = \int_0^t -\mu g dt$$

$$v = -\mu gt + v_0$$

ii. The angular velocity ω of the ring as a function of time t

$$\int_0^\omega d\omega = \int_0^t \frac{\mu g}{R} dt$$

$$\omega = \frac{\mu g}{R} t$$

(c) Derive an expression for the time it takes the ring to travel the distance L .

$$\textcircled{a} L \quad v = \omega r \quad \text{so} \quad \omega = \frac{v}{r} \quad v = -\mu g t + v_0$$

$$\omega(t) = \frac{\mu g}{r} t$$

$$\frac{v}{r} = \frac{\mu g}{r} t \quad -\mu g t + v_0 = \mu g t$$

$$v = \mu g t$$

$$2\mu g t = v_0$$

$$t = \frac{v_0}{2\mu g}$$

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L .

$$v(t) = -\mu g t + v_0$$

$$t = v_0 / 2\mu g$$

$$v(t_L) = -\mu g \left(\frac{v_0}{2\mu g} \right) + v_0$$

$$v(t_L) = v_0 - \frac{1}{2} v_0 = \frac{1}{2} v_0$$

(e) Derive an expression for the distance L .

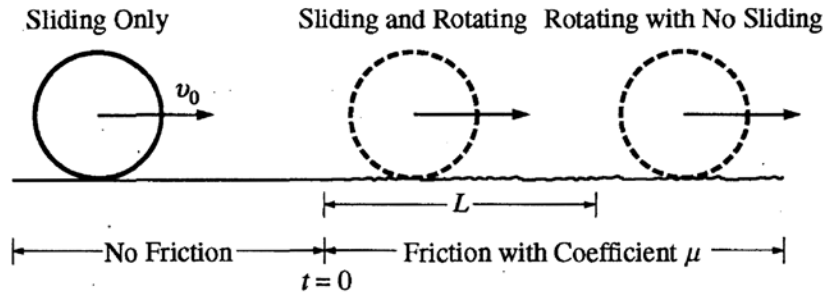
$$v_0 = v_0 \quad v_f = \frac{1}{2} v_0 \quad a = -\mu g$$

$$v^2 = v_0^2 + 2a x$$

$$\frac{1}{4} v_0^2 = v_0^2 + 2\mu g L$$

$$-\frac{3}{4} v_0^2 = -2\mu g L$$

$$L = \frac{3v_0^2}{8\mu g}$$



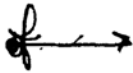
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(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$\Sigma F = ma$



$M \frac{dv}{dt} = f = \mu Mg$

$\frac{dv}{dt} = \mu g$

$f = \mu N$

$f = \mu mg$

ii. The angular velocity ω of the ring as a function of time t

$\tau = I \alpha$

$\mu Mg R = MR^2 \frac{d\omega}{dt}$

$\tau = r \times F = fR = MR^2 \cdot \frac{d\omega}{dt}$

$\frac{d\omega}{dt} = \frac{\mu g}{R}$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$\int \frac{dv}{\mu g} = \int dt$

$\frac{v}{\mu g} = t$

$v = \mu g t$

ii. The angular velocity ω of the ring as a function of time t

ω increases with rolling until $\omega R = v$

$\int d\omega = \int \frac{\mu g}{R} dt$

$\omega = \frac{\mu g t}{R}$

$\omega = \frac{\mu g t}{R} + \omega_0$

$\omega_0 = 0$ at $t = 0$

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GO ON TO THE NEXT PAGE.

(c) Derive an expression for the time it takes the ring to travel the distance L .

find t , given L , $v(t)$

$$x(t) = \int v(t) dt = x(t) = \frac{\mu g t^2}{2} + \cancel{x_0}$$

$x_0 = 0$

$$\frac{\mu g t^2}{2} = L$$

$$t^2 = \frac{2L}{\mu g}$$

$$t = \sqrt{\frac{2L}{\mu g}}$$

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L .

$$v\left(\sqrt{\frac{2L}{\mu g}}\right)$$

$$\mu g \cdot \sqrt{\frac{2L}{\mu g}} =$$

$$\sqrt{2L\mu g}$$

(e) Derive an expression for the distance L .

$$v_f^2 = v_0^2 + 2a\Delta x$$

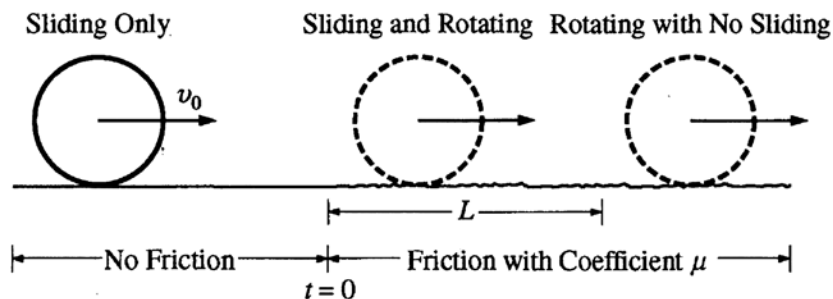
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$$a = \frac{F}{m} = \mu g$$

$$2L\mu g = v_0^2 + 2\mu g L$$

$$4\mu g L = v_0^2$$

$$L = \frac{v_0^2}{4\mu g}$$



Mech. 3.

A ring of mass M , radius R , and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L , the ring begins rolling without sliding. Express all answers to the following in terms of M , R , v_0 , μ , and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$\frac{dx}{dt} = v(t) \text{ m/s} \quad F = ma \quad F = f = \mu mg$$

$$Mmg = ma \quad a = \mu g$$

ii. The angular velocity ω of the ring as a function of time t

$$\frac{d\theta}{dt} = \omega(t) \text{ rad/s} \quad \tau = I\alpha \quad \tau = fR = I\alpha$$

$$\alpha = \frac{\tau}{I} = \frac{fR}{MR^2}$$

$$\alpha = \frac{f}{MR}$$

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

$$v_f = v_0 + at \quad m/c = m/c + m/c^2 \cdot t$$

ii. The angular velocity ω of the ring as a function of time t

$$\omega_f = \omega_0 + \alpha t \quad \text{rad/s} = \text{rad/s} + \text{rad/s}^2 \cdot t$$

- (c) Derive an expression for the time it takes the ring to travel the distance
- L
- .

$$(w_f = w_0 + at)r$$

$$v_{+f} = v_0 + a_+ t$$

~~$$v_0 = v_{+f} - a_+ t$$~~

$$v_f = v_0 + at = v_0$$

$$v_{+f} = v_0 + at + a_+ t$$

- (d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance
- L
- .

$$(w_f = w_0 + at)r$$

$$v_f = v_0 + at$$

$$v_0 = v_f - at$$

It is completely rotational now. ~~v_0~~ would be immediately after

- (e) Derive an expression for the distance
- L
- .

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

$$\Delta x = L$$

$v_f^2 =$ linear velocity
 m/s at end of L

$v_i^2 =$ linear velocity
 m/s at beginning of
 L

$a = \frac{f}{m} =$ force of
 m/s^2 friction divided
 by mass

AP[®] PHYSICS C: MECHANICS

2012 SCORING COMMENTARY

Question 3

Overview

This question assessed students' understanding of slipping and rolling motion caused by a frictional force applying a torque. It required students to evaluate both the linear motion and the rotational motion of a hoop moving across a level surface.

Sample: M3-A Score: 15

This response earned full credit. It is an excellent example of the strongest solutions. The work is well organized and easy to follow.

Sample: M3-B Score: 9

This response earned full credit in part (a)-i, even though the acceleration is shown as positive. This error will be addressed in later parts, unless it is corrected there. Full credit was earned in part (a)-ii. One point was earned in part (b)-i for setting up the integral, but the second point was lost for not including the initial velocity. In part (b)-ii full credit was awarded. No credit was earned in part (c), because the student does not indicate that $v = R\omega$. In part (d) no points were earned, because no clear equation is used. No credit was received in part (e), despite the use of a valid kinematic equation. The student incorrectly substitutes a positive acceleration; this is where the sign of the acceleration was taken into account.

Sample: M3-C Score: 4

Four points were earned in part (a); 2 points were forfeited for not expressing a or α as a derivative. No credit was received in part (b). Even though valid equations are provided, the student does not properly make substitutions into them. No points were earned in parts (c), (d), and (e). Part (e) shows a valid equation, but again, no substitutions are made.