General Notes About 2012 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded in part (b). One exception to this practice may occur in cases where the numerical answer to a later part should easily be recognized as wrong, for example, a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if the use of an equation expressing a particular concept is worth 1 point, and a student’s solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression, it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exam equation sheets. For a description of the use of such terms as “derive” and “calculate” on the exams, and what is expected for each, see “The Free-Response Sections — Student Presentation” in the AP Physics Course Description.

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of $10 \text{ m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer owing to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will eliminate the level of accuracy required to determine the difference in the numbers, and some credit may be lost.
Question 1

15 points total

(a) 3 points

For an expression of Gauss’s Law
\[ \frac{Q_i}{\varepsilon_0} = \oint \mathbf{E} \cdot d\mathbf{A} \]

For a correct intermediate step indicating that the area of the Gaussian surface is \(4\pi r^2\)
\[ \frac{Q_i}{\varepsilon_0} = E \left(4\pi r^2\right) \]

For a correct final expression, specifically using \(Q_i\)
\[ E(r) = \frac{1}{4\pi \varepsilon_0} \frac{Q_i}{r^2} \quad \text{or} \quad E(r) = \frac{kQ_i}{r^2} \]

(b) 2 points

For indicating that the enclosed charge is the sum of the inner and outer charges
\[ \frac{Q_i + Q_o}{\varepsilon_0} = \oint \mathbf{E} \cdot d\mathbf{A} \]

For a correct expression for the electric field
\[ E(r) = \frac{1}{4\pi \varepsilon_0} \frac{(Q_i + Q_o)}{r^2} \]

Note: The correct expression by itself earns both points.

(c) 2 points

For using the integral definition of potential in terms of electric field
\[ V(r) - V_\infty = -\int_\infty^r \mathbf{E} \cdot d\mathbf{r} \]

\[ V(r) = -\int_\infty^r \frac{1}{4\pi \varepsilon_0} \frac{(Q_i + Q_o)}{r^2} \, dr \]

\[ V(r) = -\frac{(Q_i + Q_o)}{4\pi \varepsilon_0} \left[ -\frac{1}{r} \right]_\infty^r \]

For the correct expression
\[ V(r) = \frac{1}{4\pi \varepsilon_0} \frac{(Q_i + Q_o)}{r} \]

Note: The correct expression by itself earns both points.
Outside the shells, the charges on each can be treated as point charges at their centers.

For using the concept of summation of point charge potentials
\[
V(r) = \frac{1}{4\pi\varepsilon_0} \sum_j \frac{q_j}{r_j}
\]

For the correct expression (the correct expression by itself earns both points)
\[
V(r) = \frac{1}{4\pi\varepsilon_0} \left( \frac{Q_i}{r} + \frac{Q_o}{r} \right)
\]

The answer from part (c), with \( Q_T = Q_i + Q_o \), can be solved for \( Q_T \). Values at the outer shell are then used to determine a numerical value.

For correct work resulting in the correct value, with units
\[
V(r) = \frac{1}{4\pi\varepsilon_0} \frac{Q_T}{r}, \ r \geq 0.20 \text{ m}
\]

\[
Q_T = 4\pi\varepsilon_0 r V(r) = \left( \frac{(0.20 \text{ m})(100 \text{ V})}{(9.0 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)} \right)
\]

\[
Q_T = 2.2 \times 10^{-9} \text{ C}
\]
Question 1 (continued)

(e) 3 points

For a segment indicating an $E$-field of 0 for $r < 0.10$ m, explicitly drawn 1 point
For a segment that is concave down and negative for $0.10$ m $< r < 0.20$ m 1 point
For a segment that is concave up and positive for $r > 0.20$ m. The line must not touch or cross the horizontal axis. 1 point

Note: The labels on the vertical axis are not to scale and are not required to receive full credit.

(f) 4 points

For a continuous set of segments that have slope discontinuities at $r = 0.10$ m and at $r = 0.20$ m 1 point
For a segment indicating a constant negative potential for $r < 0.10$ m 1 point
For a segment that is increasing, concave down, and crosses the $r$ axis, for $0.10$ m $< r < 0.20$ m 1 point
For a segment that is concave up and positive for $r > 0.20$ m. The line must not touch or cross the horizontal axis. 1 point

Note: The labels on the vertical axis are not scored and are not required to receive full credit.
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of \(-100 \, \text{V}\), and the outer shell is set at an electric potential of \(+100 \, \text{V}\), with each potential defined relative to the conventional reference point. Let \(Q_i\) and \(Q_o\) represent the net charge on the inner and outer shells, respectively, and let \(r\) be the radial distance from the center of the shells. Express all algebraic answers in terms of \(Q_i\), \(Q_o\), \(r\), and fundamental constants, as appropriate.

(a) Using Gauss’s Law, derive an algebraic expression for the electric field \(E(r)\) for \(0.10 \, \text{m} < r < 0.20 \, \text{m}\).

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0} = E \cdot A
\]

\[
\vec{E} = \frac{Q_i}{\varepsilon_0 \pi r^2} = \vec{E}(r) = \frac{Q_i}{4 \varepsilon_0 \pi r^2}
\]

(b) Determine an algebraic expression for the electric field \(E(r)\) for \(r > 0.20 \, \text{m}\).

\[
\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\varepsilon_0}
\]

\[
\vec{E} = \frac{Q_i + Q_o}{4 \varepsilon_0 \pi r^2}
\]
(c) Determine an algebraic expression for the electric potential \( V(r) \) for \( r > 0.2 \text{ m} \).

\[
V = -\int \frac{\mathbf{E}}{r} \cdot d\mathbf{r} = -\frac{kq}{r} = -\int \frac{kq}{r^2} dr
\]

\[
= -\frac{1}{4\pi \varepsilon_0} \int r^2 dr = -\frac{Q_i + Q_o}{4\pi \varepsilon_0} \left[ -\frac{1}{r} \right]_r^\infty
\]

\[
\Delta V = \frac{Q_i + Q_o}{4\pi \varepsilon_0} \left( \frac{1}{r} \right)
\]

(d) Using the numerical information given, calculate the value of the total charge \( Q_T \) on the two spherical shells \( Q_T = Q_i + Q_o \).

\[
\Delta V = \frac{Q_T}{4\pi \varepsilon_0} \left( \frac{1}{r} \right)
\]

\[
Q_T = \Delta V \cdot r - 4\pi \varepsilon_0
\]

\[
+ 100V = \frac{Q_T}{4\pi \varepsilon_0} \cdot \left( \frac{1}{r} \right)
\]

\[
(100V) \cdot (-2m) (4\pi \varepsilon_0) = Q_T = 2.2 \times 10^{-9} \text{ C}
\]

(e) On the axes below, sketch the electric field \( E \) as a function of \( r \). Let the positive direction be radially outward.

(f) On the axes below, sketch the electric potential \( V \) as a function of \( r \).
Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.

E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of −100 V, and the outer shell is set at an electric potential of +100 V, with each potential defined relative to the conventional reference point. Let \( Q_i \) and \( Q_o \) represent the net charge on the inner and outer shells, respectively, and let \( r \) be the radial distance from the center of the shells. Express all algebraic answers in terms of \( Q_i, Q_o, r, \) and fundamental constants, as appropriate.

(a) Using Gauss’s Law, derive an algebraic expression for the electric field \( E(r) \) for \( 0.10 \text{ m} < r < 0.20 \text{ m} \).

\[
\oint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside}}}{\varepsilon_0}
\]

\[
E \left( 4\pi r^2 \right) = \frac{Q_{\text{inside}}}{\varepsilon_0}
\]

\[
E = \frac{kQ_i}{r^2}
\]

(b) Determine an algebraic expression for the electric field \( E(r) \) for \( r > 0.20 \text{ m} \).

\[
\oint_A \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inside}} + Q_o}{\varepsilon_0}
\]

\[
E \left( 4\pi r^2 \right) = \frac{Q_{\text{inside}} + Q_o}{\varepsilon_0}
\]

\[
E = \frac{k(Q_i + Q_o)}{r^2}
\]
(c) Determine an algebraic expression for the electric potential \( V(r) \) for \( r > 0.20 \) m.

\[
V(r) = \frac{kQ_v}{r} + \frac{kQ_i}{r}
\]

(d) Using the numerical information given, calculate the value of the total charge \( Q_T \) on the two spherical shells \( (Q_T = Q_1 + Q_v) \).

\[
\frac{V}{\partial \rho} = \frac{K(Q_i + Q_v)}{r^2} = -100
\]

\[
\frac{K(Q_i + Q_v)}{r} = -100
\]

\[
\frac{KQ_i}{r} = -100
\]

\[
Q_i + Q_v = \frac{-100}{K}
\]

\[
Q_i = -\frac{100}{K}
\]

\[
Q_v = \alpha \lambda \times 10^{-9}
\]

(e) On the axes below, sketch the electric field \( E \) as a function of \( r \). Let the positive direction be radially outward.

(f) On the axes below, sketch the electric potential \( V \) as a function of \( r \).
E&M. 1.

Two thin, concentric, conducting spherical shells, insulated from each other, have radii of 0.10 m and 0.20 m, as shown above. The inner shell is set at an electric potential of \(-100 \text{ V}\), and the outer shell is set at an electric potential of \(+100 \text{ V}\), with each potential defined relative to the conventional reference point. Let \(Q_i\) and \(Q_o\) represent the net charge on the inner and outer shells, respectively, and let \(r\) be the radial distance from the center of the shells. Express all algebraic answers in terms of \(Q_i\), \(Q_o\), \(r\), and fundamental constants, as appropriate.

(a) Using Gauss's Law, derive an algebraic expression for the electric field \(E(r)\) for \(0.10 \text{ m} < r < 0.20 \text{ m}\).

\[
\oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

\[
E = \frac{Q_i}{4\pi \varepsilon_0 r^2}
\]

(b) Determine an algebraic expression for the electric field \(E(r)\) for \(r > 0.20 \text{ m}\).

\[
\oint E \cdot dA = \frac{Q_{\text{enclosed}}}{\varepsilon_0}
\]

\[
E = \frac{Q_o}{4\pi \varepsilon_0 r^2}
\]
(c) Determine an algebraic expression for the electric potential $V(r)$ for $r > 0.20 \text{ m}$.

$$V = \frac{Q_0 r}{4 \pi r \varepsilon_0}$$

(d) Using the numerical information given, calculate the value of the total charge $Q_T$ on the two spherical shells ($Q_T = Q_1 + Q_2$).

- For inner shell: $V = \frac{kq}{r}$
  
  $$-100 = \left( \frac{9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}}{} \right) (Q_1)$$
  
  $Q_1 = -1.11 \times 10^{-9} \text{ C}$

- For outer shell: $V = \frac{kq}{r}$
  
  $$100 = \left( \frac{9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}}{} \right) (Q_0)$$
  
  $Q_0 = 2.22 \times 10^{-9} \text{ C}$

(e) On the axes below, sketch the electric field $E$ as a function of $r$. Let the positive direction be radially outward.

(f) On the axes below, sketch the electric potential $V$ as a function of $r$.
Question 1

Overview

This question assessed students’ understanding of the concepts of electric charge, electric field strength, and electric potential, and these concepts’ interdependency in the given two-concentric-conducting-sphere system at various locations.

Sample: E1-A
Score: 13

Full credit was earned in parts (a) through (d). Gauss’s Law is properly used to derive expressions for the electric field. The potential difference and total charge are correct. One point was lost in part (e) because the middle segment of the graph should be negative. Another point was lost in part (f) because the middle segment should not be linear.

Sample: E1-B
Score: 9

In part (a) 2 points were awarded. Although the answer is correct, 1 point was lost for the incorrect expression of Gauss’s Law. Full credit was earned in part (b); the student had already been penalized for writing Gauss’s Law incorrectly in part (a). One point was earned in part (c) for beginning the answer with the idea of a summation of point charge potentials. No credit was awarded in part (d) because the sign is incorrect and there are no units on the answer. In part (e) full credit was earned. Only 1 point was earned in part (f) for the correct final segment of the graph.

Sample: E1-C
Score: 6

Full credit was earned in part (a). No credit was awarded in part (b), because the expression for the enclosed charge is incorrect. Nor were any points earned in part (c), because the student does not integrate or attempt to sum up point charge potentials. The answer in part (d) is incorrect and earned no credit. (Note that the student does have the correct answer for the total charge $Q_t$, but not for $Q_r$.) One point was awarded in part (e) for the correct final segment of the graph. In part (f) 2 points were earned for the correct first and last segments, but the middle segment is incorrect, and the graph is not continuous.