

AP[®] STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) describe a situation as a series of Bernoulli trials; (2) calculate probabilities of events involving Bernoulli trials; (3) recognize whether or not an event is likely to occur.

Solution

Part (a):

Let Y denote the number of flights Sam must make until he receives his first upgrade. The random variable Y follows a geometric distribution with $p = 0.1$. The probability that Sam's upgrade will occur after his third flight is calculated below.

$$\begin{aligned} P(Y \geq 4) &= 1 - P(Y \leq 3) \\ &= 1 - [P(Y = 1) + P(Y = 2) + P(Y = 3)] \\ &= 1 - [0.1 + 0.9 \times 0.1 + (0.9)^2 \times 0.1] \\ &= 1 - [0.1 + 0.09 + 0.081] \\ &= 0.729 \end{aligned}$$

Part (b):

Let p denote the probability that Sam will be upgraded to first class on a particular flight. Let X denote the number of upgrades Sam will receive in 20 flights. The random variable X follows a binomial distribution with $n = 20$ independent trials and $p = 0.1$. The probability that Sam will be upgraded exactly 2 times in his next 20 flights is calculated as follows.

$$\begin{aligned} P(X = 2) &= \binom{20}{2} (0.1^2)(0.9^{18}) \\ &\approx 0.2852 \end{aligned}$$

Part (c):

Let X denote the number of upgrades Sam will receive in 104 flights. The random variable X follows a binomial distribution with $n = 104$ independent trials and $p = 0.1$. Thus,

$$\begin{aligned} P(X > 20) &= 1 - P(X \leq 20) \\ &\approx 1 - 0.9986 \\ &\approx 0.0014. \end{aligned}$$

Because this probability is so small, it is very unlikely that Sam would receive more than 20 upgrades in 104 flights if the airline's claim is correct. This would be expected to happen less than 1 percent of the time, indicating that one should be surprised if Sam receives more than 20 upgrades during the next year.

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Question 3 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes a correct probability and shows supporting work.

Note: An alternative solution to part (a) is:

$$\begin{aligned} P(\text{first upgrade after third flight}) &= \\ P(\text{no upgrade on (flight 1 or flight 2 or flight 3)}) &= \\ P(\text{no upgrade on flight 1 and no upgrade on flight 2 and no upgrade on flight 3}) &= \\ &= 0.9 \times 0.9 \times 0.9 = 0.729 \end{aligned}$$

Partially correct (P) if the response includes a correct calculation of a related probability such as $P(Y = 3) = (0.9)(0.9)(0.1)$ or $P(Y = 4) = (0.9)(0.9)(0.9)(0.1)$, with supporting work, *OR* the response includes a correct probability with insufficient supporting work.

Note: The calculator command, $1 - \text{geomtcdf}(0.1, 3)$ with no other justification should be scored no higher than P, unless the value 0.1 is labeled as p somewhere in the response, in which case a score of E can be given.

Incorrect (I) if an answer is provided with no supporting work, *OR* an unreasonable probability (greater than 1 or less than 0) is provided.

Part (b) is scored as follows:

Essentially correct (E) if the response includes a correct probability and shows supporting work.

Partially correct (P) if the response includes supporting work but makes a calculation mistake that results in a reasonable incorrect probability. For example, $0.1^2 \times 0.9^{18}$ should be scored as partially correct, *OR* if the response includes a correct probability with insufficient supporting work.

Notes

- The calculator command, $\text{binomialpdf}(20, 0.1, 2)$, with no other justification should be scored no higher than P, unless somewhere in the response, the value 0.1 is labeled as p and the value of 20 is labeled as n . In such a case, a score of E can be given for this part.
- If the response in part (a) was downgraded to P for failure to identify the geometric parameter, do not downgrade part (b) for this same oversight.

Incorrect (I) if an answer is provided with no supporting work, *OR* an unreasonable probability (greater than 1 or less than 0) is provided.

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Question 3 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response includes an answer to the question that is linked to a correct probability.

Partially correct (P) for any of the following:

- The response includes an answer to the question *AND* provides a justification based on expected value.
- The response includes an answer to the question that is linked to an incorrect but reasonable probability.
- The response gives a correct probability *AND* includes a correct answer to the question but fails to link the two.
- The response gives a correct probability *BUT* fails to answer the question.

Incorrect (I) if the response fails to meet the criteria for E or P.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect

3A

3A

3. An airline claims that there is a 0.10 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Sam is a frequent flier who always purchases coach-class tickets.
- (a) What is the probability that Sam's first upgrade will occur after the third flight?

$$\begin{aligned}
 &P(\text{Sam's upgrade did not happen during the first three flights}) \\
 &= \frac{1}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \\
 &= 0.729
 \end{aligned}$$

- (b) What is the probability that Sam will be upgraded exactly 2 times in his next 20 flights?

$$\begin{aligned}
 &P(\text{Sam will be upgraded exactly 2 times in next 20 flights}) \\
 &= {}^{20}C_2 (0.1)^2 (0.9)^{18} \\
 &= 0.285
 \end{aligned}$$

- (c) Sam will take 104 flights next year. Would you be surprised if Sam receives more than 20 upgrades to first class during the year? Justify your answer.

Yes. With the probability of 0.1, Sam should get around 10 upgrades.

Since $P(\text{Sam getting 20 or less upgrades}) = 0.9986$,
 the probability of him getting more than 20 upgrades is $1 - 0.9986$
 $= 0.0014$

There is very slim chance of getting more than 20 upgrades.

GO ON TO THE NEXT PAGE.

3B

$p = 0.1$

3B

3. An airline claims that there is a 0.10 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Sam is a frequent flier who always purchases coach-class tickets.

- (a) What is the probability that Sam's first upgrade will occur after the third flight?

Geometric setting: $p = 0.1$

$$P(X > 3) = 1 - P(X \leq 2) = 1 - \{0.1 + (0.9)(0.1) + (0.9)^2(0.1)\}$$

$$= \boxed{0.729}$$

* assumptions:

- ① Independent outcome
- ② fixed # of success.
- ③ count # of trials till 1st success

- (b) What is the probability that Sam will be upgraded exactly 2 times in his next 20 flights?

Binomial setting: $B(20, 0.1)$

$$P(X = 2) = {}_{20}C_2 \cdot (0.1)^2 (0.9)^{18}$$

$$= \boxed{0.28517}$$

* assumptions:

- ① Independent
- ② fixed # of trials
- ③ count # of success.

- (c) Sam will take 104 flights next year. Would you be surprised if Sam receives more than 20 upgrades to first class during the year? Justify your answer.

Binomial: $B(104, 0.1)$ * assumptions checked above.

$$P(X > 20) = 1 - P(X \leq 19)$$

$$= 1 - 0.9968 = \boxed{0.0032}$$

→ The probability of Sam receiving more than 20 upgrades in 104 flights is only 0.003, which is highly unlikely. Thus, if he gets so, I would be surprised.

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3C

3C

3. An airline claims that there is a 0.10 probability that a coach-class ticket holder who flies frequently will be upgraded to first class on any flight. This outcome is independent from flight to flight. Sam is a frequent flier who always purchases coach-class tickets.

(a) What is the probability that Sam's first upgrade will occur after the third flight?

$$P = (0.9)^3 \times 0.1 = 0.0729$$

So the probability is 7.29%

(b) What is the probability that Sam will be upgraded exactly 2 times in his next 20 flights?

$$P = \binom{20}{2} (0.1)^2 (0.9)^{18} = 0.28518$$

So the probability is 28.5%

(c) Sam will take 104 flights next year. Would you be surprised if Sam receives more than 20 upgrades to first class during the year? Justify your answer.

The expected value = $104 \times 0.1 = 10.4$

Since $20 > 10.4$,

I will be surprised if Sam receives more than 20 upgrades.

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AP[®] STATISTICS
2011 SCORING COMMENTARY (Form B)

Question 3

Sample: 3A

Score: 4

In part (a) the response includes the correct probability, along with adequate supporting work, that Sam's first upgrade will occur after the third flight. Part (a) was scored as essentially correct. In part (b) the response includes the binomial formula and the substitution of correct numerical values. Part (b) was scored as essentially correct. The response in part (c) begins with the correct answer ("Yes") to the question and follows with the correct, but unnecessary, statement that if the probability of an upgrade is 0.1 for each flight, "Sam should get around 10 upgrades." The response includes the correct probability ("0.0014") that Sam will receive more than 20 upgrades, along with the observation that the probability is "very slim." Although the response would be stronger if the answer were linked more closely to the observation that the probability of more than 20 upgrades is small, part (c) was scored as essentially correct. Because three parts were scored as essentially correct, the response earned a score of 4.

Sample: 3B

Score: 3

In part (a) the response includes a correct calculation of the probability that Sam's first upgrade will occur after the third flight and shows the work using the geometric distribution. Part (a) was scored as essentially correct. In part (b) the response includes the binomial formula with the correct numerical values substituted. Part (b) was also scored as essentially correct. This response includes assumptions in both parts (a) and (b). Although it is not necessary to state assumptions in this question, the lists include no incorrect information. The calculation given in part (c) omits the probability of receiving exactly 20 upgrades. Therefore, calculated probability is actually the probability of receiving at least 20 upgrades, rather than the probability of receiving more than 20 upgrades. The response correctly answers the question, with linkage to this incorrect (but reasonable) probability. Part (c) was thus scored as partially correct. Because two parts were scored as essentially correct and one part was scored as partially correct, the response earned a score of 3.

Sample: 3C

Score: 2

The response in part (a) shows the calculation of the related probability that the first upgrade occurs on the fourth flight, rather than the requested probability that the first upgrade occurs after the third flight, so part (a) was scored as partially correct. In part (b) the response includes the binomial formula with the correct numerical values substituted, and part (b) was scored as essentially correct. In part (c) the response includes a correct answer to the question, along with a justification based on the expected number of upgrades. Although it is true that 20 upgrades is more than the expected 10.4 upgrades if the probability of an upgrade is 0.1 for each flight, the observation does not indicate *how* surprising 20 upgrades would be. For that reason, this justification was not considered to be as strong as a justification based on the probability of more than 20 upgrades, and part (c) was scored as partially correct. Because one part was scored as essentially correct and two parts were scored as partially correct, the response earned a score of 2.