CollegeBoard AP

Student Performance Q&A:

2011 AP® Calculus AB and Calculus BC Free-Response Questions

The following comments on the 2011 free-response questions for AP® Calculus AB and Calculus BC were written by the Chief Reader, Michael Boardman of Pacific University in Forest Grove, Ore. They give an overview of each free-response question and of how students performed on the question, including typical student errors. General comments regarding the skills and content that students frequently have the most problems with are included. Some suggestions for improving student performance in these areas are also provided. Teachers are encouraged to attend a College Board workshop to learn strategies for improving student performance in specific areas.

Question AB1

What was the intent of this question?

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 6$. The position, $x(t)$, of the particle is unknown, but velocity and acceleration functions, $v(t)$ and $a(t)$, respectively, are provided. Part (a) asked students whether the speed of the particle is increasing or decreasing at time $t = 5.5$. Students should have evaluated both the velocity and the acceleration functions at $t = 5.5$; because $v(5.5) < 0$ and $a(5.5) < 0$, the particle's speed is increasing. Part (b) asked for the average velocity of the particle during the given time interval. This can be computed as an average value, $\frac{1}{6-0}\int_0^6 v(t)\,dt$, and evaluated on a calculator. Part (c) asked for the total distance traveled by the particle. The total distance is the value of $\int_0^6 |v(t)|\,dt$, which can be computed directly on the calculator, or by splitting the interval into a segment on which $v(t) > 0$ and one on which $v(t) < 0$, and then appropriately combining the corresponding definite integrals of velocity. Part (d) highlighted that the particle changes direction exactly once during the interval and asked for the position of the particle at that time. If they had not already done so, students should have used their calculators to find the solution to $v(t) = 0$ with $0 \le t \le 6$. If the solution is $t = A$, the position of the particle at that time is then calculated as $x(A) = 2 + \int_0^A v(t)\,dt$.

How well did students perform on this question?

Overall, student performance on this question was disappointing. The mean score was 2.79 out of a possible 9 points. About 2.5 percent of students earned all 9 points. About 21 percent did not earn any points.

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In general, students did not do well on part (a), where many did not earn either of the possible points. Most students correctly set up and evaluated the average value integral to earn both points in part (b). In part (c) many students failed to earn either of the possible points. However, students who successfully set up the integral for total distance, earning the first point, usually earned the answer point as well. In part (d) few students earned all 3 points.

What were common student errors or omissions?

Some students had difficulty correctly entering the functions for velocity and acceleration into their calculators. Many responses had answers consistent with either $v(t) = 2\sin\left(e^{t/4}\right)$, omitting the "+1," or $v(t) = 2\sin\left(e^{t/4} + 1\right)$, misplacing the closing parentheses on the sine function. In part (a) many students considered only the sign of either the velocity or the acceleration, found it to be negative, and erroneously concluded that the speed was decreasing. In part (b) some students computed average acceleration, $\dfrac{v(6) - v(0)}{6 - 0}$, rather than average velocity. In part (c) the most common error was to neglect the change in direction of the particle, computing $\int_0^6 v(t)\,dt$ instead of the correct $\int_0^6 |v(t)|\,dt.$ In part (d) many students found the time $t = 5.196$ at which the particle changed direction but did not continue on to find the position of the particle at that time. Others attempted to find the position but did not use the initial position $x(0) = 2.$

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

In particle motion problems, it is important for students to be able to distinguish between speed and velocity, between average velocity and average acceleration, and between total distance traveled and displacement. Also, students should carefully check any expressions entered into their calculators to be sure they have all parentheses correctly placed. Students need practice finding the position of a particle at a particular time both from determining the position function and from using the Fundamental Theorem of Calculus $x(b) = x(a) + \int_a^b v(t)\,dt.$

Question AB2/BC2

What was the intent of this question?

In this problem students were presented with a table giving Celsius temperatures $H(t)$ of a cooling pot of tea during selected times between $t = 0$ and $t = 10$ minutes. Part (a) asked for an approximation for the rate of change of the tea's temperature at time $t = 3.5.$ Students needed to construct a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that was supported by the table. Part (b) asked for an interpretation of $\frac{1}{10}\int_0^{10} H(t)\,dt$ and a numeric approximation to this expression using a trapezoidal sum with the four intervals indicated by the table. Students should have recognized this expression as providing the average temperature of the tea, in degrees Celsius, across the

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time interval $0 \le t \le 10$ minutes. Part (c) asked for an evaluation and interpretation of $\int_0^{10} H'(t)\,dt$. Students needed to apply the Fundamental Theorem of Calculus and use values from the table to compute $H(10) - H(0)$. In part (d) students were told about biscuits that were removed from an oven at time $t = 0$. It was given that the biscuits' temperature was $100°\text{C}$ initially, and that a function $B(t)$ modeling the temperature of the biscuits has derivative $B'(t) = -13.84e^{-0.173t}$. Students were asked how much cooler the biscuits are than the tea at time $t = 10$ minutes. This was answered by taking the difference between the tea's temperature, $H(10)$, as supplied by the table, and the biscuits' temperature, $B(10)$, computed by $B(10) = 100 + \int_0^{10} B'(t)\,dt$.

How well did students perform on this question?

Student performance on this question was average. The mean score was 3.31 for AB students and 5.48 for BC students out of a possible 9 points. About 5.5 percent of AB students and 18 percent of BC students earned all 9 points. About 23 percent of AB students and 6.5 percent of BC students did not earn any points.

Most students earned the point in part (a). In parts (b) and (c) many students had difficulty giving the meaning of the expression. In part (b) some did not earn the trapezoidal estimate point. Of those who did, some made subsequent errors and did not earn the third point. In part (d) students who knew to use a definite integral generally earned the first 2 points. Some students earned only the first point in part (d) by attempting, but not succeeding with, a solution involving an indefinite integral.

What were common student errors or omissions?

Explaining the meanings of the expressions in parts (b) and (c) proved difficult for many students. Some confused the average temperature and change in temperature with the average rate of change of temperature. In part (b) many students mistakenly applied the Trapezoidal Rule, failing to recognize that the time intervals were not of equal lengths. In part (c) some students did not apply the Fundamental Theorem of Calculus in order to evaluate $\int_0^{10} H'(t)\,dt$. In part (d) some students found the temperature of the biscuits but did not give a final answer to the question.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need practice using the Fundamental Theorem of Calculus in its various forms, including $f(b) = f(a) + \int_a^b f'(t)\,dt$. Although being able to apply formulas for integral approximations is a useful skill, it is important for students to understand the underlying development of these formulas so that they can apply the general technique in a variety of situations. Students also need to be able to interpret various mathematical objects in the context of applications, which includes the appropriate use of units.

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Question AB3

What was the intent of this question?

This problem involved the graphs of functions $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$ that enclose a region R in the first quadrant. A figure depicting R is supplied, with the label $\left(\frac{1}{2}, 1\right)$ at the point of intersection of the graphs of f and g. Part (a) asked for an equation of the line tangent to the graph of f at $x = \frac{1}{2}$. Part (b) asked for the area of R, which required students to set up and evaluate an appropriate definite integral. For part (c) students needed to provide an integral expression for the volume of the solid that is generated when R is rotated about the horizontal line $y = 1$.

How well did students perform on this question?

Student performance on this question was good, particularly in parts (a) and (b). The mean score was 4.64 out of a possible 9 points. About 8 percent of students earned all 9 points. About 10 percent did not earn any points.

Most students earned both points in part (a). Most students earned the setup point in part (b) and continued on to earn at least 1of the 2 antiderivative points. Part (c) was the most difficult, with many students earning at most 1 point.

What were common student errors or omissions?

In part (b) many students had difficulty antidifferentiating $\sin(\pi x)$. Some students attempted to find the area of R by computing $\int_0^{1/2} (f(x) - g(x))\, dx$, which gives the negative of the actual area. These students were eligible to earn the answer point if they correctly gave the positive area without making an incorrect statement about the aforementioned integral giving a positive number. In part (c) some students wrote expressions that were incorrect because of the placement of parentheses. Other students failed to write the square of the appropriate terms in the integrand.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Although being able to apply formulas for volumes of revolution is useful, it is critical that students understand the underlying development of the formulas from volumes of solids with known cross-sectional areas. This will enable students to apply the general method in a variety of situations. Students must be careful in their use of mathematical notation. Missing or misplaced symbols and poor presentation of otherwise good ideas can lead to ambiguity or incorrect results. Students are also expected to show intermediate work and final answers in complete, correct form.

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Question AB4/BC4

What was the intent of this question?

This problem provided the graph of a continuous function *f*, defined for $-4 \le x \le 3.$ The graph consisted of two quarter circles and one line segment. The function g was defined by $g(x) = 2x + \int_0^x f(t)\,dt.$ Part (a) asked for $g(-3)$, an expression for $g'(x),$ and the value of $g'(-3).$ These items tested the interpretation of a definite integral in terms of the area of a region enclosed by the *x*-axis and the graph of the function given in the integrand, as well as the application of the Fundamental Theorem of Calculus to differentiate a function defined by an integral with a variable upper limit of integration. Part (b) asked for the *x*-coordinate of the point at which g attains an absolute maximum for $-4 \le x \le 3.$ Several approaches were possible, but they all begin with identification of candidates using the expression for $g'(x)$ found in part (a). Part (c) asked for locations of points of inflection for the graph of g, involving another analysis of $g'(x).$ Part (d) asked for the average rate of change of *f* on $-4 \le x \le 3$ and tested knowledge of the hypotheses of the Mean Value Theorem to explain why that theorem is not contradicted given the fact that its conclusion does not hold for *f* on $-4 \le x \le 3.$

How well did students perform on this question?

Student performance on this question was below expectation. The mean score was 2.44 for AB students and 4.13 for BC students out of a possible 9 points. About 0.4 percent of AB students and 1.6 percent of BC students earned all 9 points. About 29.5 percent of AB students and 8 percent of BC students did not earn any points.

Many students earned only 1 of the 3 possible points in part (a). In part (b) some earned the first 2 points but failed to give a justification for an absolute maximum and did not earn the third point. Few students earned the point in part (c). Many students earned the first point in part (d), correctly finding the average rate of change of *f* on the interval, but most did not earn the second point.

What were common student errors or omissions?

In part (a) many students did not correctly handle the fact that the lower limit of integration was larger than the upper limit of integration. Some students who correctly dealt with the $2x$ term when evaluating $g(-3)$ ignored it when taking the derivative for the second point. Some mistakenly found $g(3)$ and $g'(3)$ rather than $g(-3)$ and $g'(-3).$ In part (b) many students gave justification for a local maximum rather than a global maximum. In part (c) many students erroneously listed $x = -3$ as the location of a point of inflection because that is a location where $g''(x) = 0.$ Others mistakenly ruled out $x = 0$ as a point of inflection point because $g''(x)$ did not exist at $x = 0.$ In part (d) most students had difficulty expressing why the Mean Value Theorem did not apply to *f* on the interval.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

It is important for students to understand a variety of methods for determining when a local maximum is a global maximum, and to understand the difference between a local argument and a global argument.

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Students need practice communicating mathematics using clear, mathematically correct, and mathematically precise language. Students also need practice and facility with functions whose definitions include an integral, such as $\int_0^x f(t)\,dt$. They need to be able to reason from the graph of a function, the graph of its derivative, or a function whose definition involves a presented graph.

Question AB5/BC5

What was the intent of this question?

The context of this problem was accumulating waste at a landfill. The landfill contained 1400 tons of waste at the beginning of 2010, and a function W modeling the total tons of waste in the landfill satisfies $\frac{dW}{dt} = \frac{1}{25}(W - 300)$, where t is measured in years since the start of 2010. Part (a) asked for an approximation to $W\left(\frac{1}{4}\right)$ using a tangent line approximation to the graph of W at $t = 0$. Part (b) asked for $\frac{d^2W}{dt^2}$ in terms of W, and students should have used a sign analysis of $\frac{d^2W}{dt^2}$ to determine whether the approximation in part (a) is an overestimate or an underestimate. Part (c) asked students to solve the initial value problem $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with $W(0) = 1400$ to find $W(t)$. Students should have used the method of separation of variables.

How well did students perform on this question?

Overall, student performance on this question was very poor, with about 54 percent of AB students and 27 percent of BC students earning no points. However, among students who did score points, performance was average. The mean score was 1.63 for AB students and 3.53 for BC students out of a possible 9 points. About 0.5 percent of AB students and 2.4 percent of BC students earned all 9 points.

Those students who earned points generally earned both points in part (a). Most students failed to earn either point in part (b). Part (c) was a standard question. Students who earned points generally earned either all 5 points or just the second, third and fourth points, having made an error in separating the variables.

What were common student errors or omissions?

Throughout the problem, many students made algebra and arithmetic errors in their computations. In part (a) many students wrote that quantities were equal when they were not. For example, it was common to see "$\left(\frac{1}{25}\right)(1100) = 44\left(\frac{1}{4}\right) = 11 + 1400 = 1411.$" In part (b) nearly all students wrote that $\frac{d^2W}{dt^2} = \frac{1}{25}$, which is incorrect. Some students who correctly computed $\frac{d^2W}{dt^2}$ argued that their estimate from part (a) was an underestimate by stating that $\frac{d^2W}{dt^2} > 0$ only at a single point, $t = 0$, rather than on the entire

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interval. In part (c) many students made algebra errors when separating variables. Many of these students did not have $\int \frac{1}{W-300}\,dW$ as one of the integrals to evaluate.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need practice with differential equations in contextual problems. It is important to emphasize that much can be observed about the behavior of solutions to a differential equation without actually solving the equation. Students also need practice with autonomous differential equations, those equations involving only the unknown function and its derivative, but not the independent variable. Students should be aware that determining whether an estimate is larger or smaller than the actual value of a function generally requires knowledge of the behavior of the function and its derivatives on an entire interval, not just at a single point.

Question AB6

What was the intent of this question?

This problem defined the function f using one expression for $x \le 0$ and a different expression for $x > 0$. Part (a) asked whether f is continuous at $x = 0$. Students needed to acknowledge that the left- and right-hand limits as $x \to 0$ and the value $f(0)$ all agree. Part (b) asked for a piecewise expression for $f'(x)$ and the value of x for which $f'(x) = -3$. This involved taking the symbolic derivatives of the branches of f and recognizing which piece produces a value of -3. Part (c) asked for the average value of f on the interval $[-1, 1]$. The required integral must be split at 0 to use the antiderivatives of the two branches of f.

How well did students perform on this question?

Students performed relatively well on this question, particularly given that it involved a piecewise-defined function and was the last question on the exam. The mean score was 3.03 out of a possible 9 points. About 0.5 percent of students earned all 9 points. About 18.5 percent of students did not earn any points.

In part (a) most students earned 1 of the 2 possible points. In part (b) many students differentiated the expressions for f but did not express $f'(x)$ as a piecewise-defined function, earning 1 of the 2 derivative points. Few students earned the third point. In part (c) many students earned the first point and 1 of the 2 possible antiderivative points. Students who earned the first 3 points generally earned the fourth as well.

What were common student errors or omissions?

In part (a) most students tried to show that the function was continuous by evaluating each expression at $x = 0$. This earned 1 of the 2 possible points. To earn both points, students needed to use limits to show an understanding of continuity. In part (b) although many students were able to give the derivatives of each expression comprising f, most did not write their derivatives as a piecewise-defined function. Many students attempted to solve $-2\cos x = -3$ and mistakenly produced a value for x from this equation. In

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part (c) some students computed the average rate of change of f, computing $\frac{f(1) - f(-1)}{2}$ rather than the correct $\frac{1}{2}\int_{-1}^{1} f(x)\,dx.$ Among those having the correct integral, many earned the first point and 1 of the 2 antiderivative points. The fourth point proved difficult to attain.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

It is important that students use limits to show that a function f is continuous at $x = a.$ In the case of a piecewise-defined function, students must show that $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a).$ Piecewise-defined functions offer a rich context in which to emphasize the ideas of continuity and differentiability. They also reveal the importance of mathematical notation. Paper-and-pencil techniques for differentiation and antidifferentiation remain critical skills for calculus.

Question BC1

What was the intent of this question?

This problem described the path of a particle whose motion is described by $(x(t), y(t)),$ where $x(t)$ and $y(t)$ satisfy $\frac{dx}{dt} = 4t + 1$ and $\frac{dy}{dt} = \sin\left(t^2\right)$ for $t \ge 0.$ It was also given that $x(0) = 0$ and $y(0) = -4.$ Part (a) asked for the speed of the particle at time $t = 3$ and the acceleration vector of the particle at $t = 3.$ For part (b) students needed to recognize that the slope of the line tangent to the particle's path at $t = 3$ is given by $\left.\frac{dy}{dx}\right|_{t=3} = \left.\frac{dy/dt}{dx/dt}\right|_{t=3},$ a consequence of the chain rule. Part (c) asked for the position of the particle at time $t = 3.$ This required two applications of the Fundamental Theorem: $x(3) = x(0) + \int_0^3 \frac{dx}{dt}\,dt$ and $y(3) = y(0) + \int_0^3 \frac{dy}{dt}\,dt.$ Part (d) asked for the total distance traveled by the particle over the time interval $0 \le t \le 3.$ This is found by integrating $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ over the interval $0 \le t \le 3.$

How well did students perform on this question?

Student performance on this question was very good. The mean score was 5.24 out of a possible 9 points. About 14 percent of the students earned all 9 points. About 5 percent did not earn any points.

Most students earned both points in part (a) and the point in part (b). In part (c) many students earned the 2 points for the x-coordinate but did not earn the points for the y-coordinate. Students who earned the first point in part (d) generally earned the second point as well.

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What were common student errors or omissions?

In part (a) some students mistakenly computed $\frac{y'(3)}{x'(3)}$ as the speed of the particle at time $t = 3$. In part (b) some students gave the speed as the slope of the tangent line. In part (c) many tried to antidifferentiate $\sin\left(t^2\right)$ rather than use their calculators to evaluate the definite integral. Some students did not use $y(0) = -4$ in their calculations of the position of the particle at time $t = 3$. In part (d) many students incorrectly attempted to use the arc length formula $\int_0^3 \sqrt{1 + (f'(x))^2}\, dx$ for the graph of a function $y = f(x)$, rather than the arc length formula for a parametric curve.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need considerable practice working with parametric motion, including computations of speed, velocity and distance traveled. The Fundamental Theorem of Calculus written in the form $f(b) = f(a) + \int_a^b f'(t)\, dt$ is one of the most useful results of calculus. Students are expected to present intermediate work and final answers in complete, correct form. Careful attention to writing complex expressions, especially those containing parentheses, will help students avoid some common errors.

Question BC3

What was the intent of this question?

This problem defined a region R in the first quadrant bounded by the graph of $f(x) = e^{2x}$, the coordinate axes, and the line $x = k$, where $k > 0$. A figure depicting R was provided. Part (a) asked for an expression involving an integral that gives the perimeter of R in terms of k. This is the sum of the lengths of the three straight-segment sides and a definite integral giving the length of the curve $y = f(x)$ from $x = 0$ to $x = k$. Part (b) asked for the volume V, in terms of k, of the solid that results from rotating region R about the x-axis. Students needed to set up and evaluate an integral involving the parameter k. Part (c) required students to use the chain rule to find $\frac{dV}{dt}$ when $k = \frac{1}{2}$, given that $\frac{dk}{dt} = \frac{1}{3}$.

How well did students perform on this question?

Students performed very well on this problem, with about 19.5 percent earning all 9 points. This is remarkable given the presence of the parameter along with the exponential function. The mean score was 5.51 out of a possible 9 points. About 4 percent of students did not earn any points.

In part (a) nearly half the students earned all 3 points. Others generally earned either no points or 1 point, specifically the derivative point. In part (b) most students earned the first 2 points. The antiderivative and answer points were more difficult for students. Part (c) proved to be the most difficult, with many students making errors in the computation of the derivative. Students were able to use either their final

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answer from part (b) or the integral setup from part (b) and the Fundamental Theorem of Calculus to compute the rate of change of volume. Both were valid approaches to the problem.

What were common student errors or omissions?

The most common errors in part (a) were an incorrect integral for arc length, no integral for arc length, and neglecting other parts of the boundary of R. In part (b) some students wrote an integral expression for volume but did not evaluate the integral. Some did not correctly antidifferentiate e^{4x}. Many students did not correctly apply the chain rule in their derivative computation in part (c). Some did not differentiate e^{4x} correctly.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students need practice differentiating and integrating functions of the form $f(x) = e^{ax}$. They need to clearly communicate the work they have done to arrive at an answer. They also need to be reminded to show intermediate steps as part of their solutions. The terminating differential in an integral plays an important role, both to separate the integral from the remaining expression and to communicate which variable is the variable of integration.

Question BC6

What was the intent of this question?

The series problem defined $f(x) = \sin\left(x^2\right) + \cos x$ and provided a graph of $y = \left|f^{(5)}(x)\right|$. Parts (a) and (b) concerned series manipulations. Part (a) asked for the first four nonzero terms of the Taylor series for $\sin x$ about $x = 0$ and also for the first four nonzero terms of the Taylor series for $\sin\left(x^2\right)$ about $x = 0$. Part (b) asked for the first four nonzero terms of the Taylor series for $\cos x$ about $x = 0$ and also for the first four nonzero terms of the Taylor series for $f(x)$ about $x = 0$. Part (c) asked for the value of $f^{(6)}(0)$. Although an energetic student could have started by computing the sixth derivative of f, it was expected that students would have recognized that the coefficient of x^6 in the Taylor series for $f(x)$ about $x = 0$ is $\frac{f^{(6)}(0)}{6!}$. Part (d) tested the Lagrange error bound for $P_4(x)$, the fourth-degree Taylor polynomial for f about $x = 0$. Students needed to acquire a correct and sufficient bound on $\left|f^{(5)}(x)\right|$ for $0 \le x \le \frac{1}{4}$ from the supplied graph and use this bound to verify that $\left|P_4\left(\frac{1}{4}\right) - f\left(\frac{1}{4}\right)\right| < \frac{1}{3000}$.

How well did students perform on this question?

Overall, students performed better on this series problem than on others in recent years. The mean score was 3.97 out of a possible 9 points. About 1 percent of students earned all 9 points. About 13 percent did not earn any points.

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Most students earned both points in part (a). In part (b) most students earned the first point for the series for cosine, but many earned at most 1 of the 2 possible points for the series for $f(x)$. Not many earned the point in part (c). In part (d) students who earned the first point often earned the second point as well.

What were common student errors or omissions?

Some students did not correctly recall the series for $\sin x$ or for $\cos x$. Many showed a lack of understanding of series manipulation, particularly in substituting x^2 for x in the series for $\sin x$ to obtain the series for $\sin\left(x^2\right)$. Some students did not combine the like terms in part (b) in order to present the first four nonzero terms of the requested Taylor series for *f*. Many students did not use the coefficient of the x^6 term in order to determine the value of $f^{(6)}(0)$. Many also did not know the form of the Lagrange error bound for Taylor polynomials. Among those who did, some had difficulty finding a correct value for $\max\limits_{0 \le x \le \frac{1}{4}} \left| f^{(5)}(x) \right|$ from the graph.

Based on your experience of student responses at the AP Reading, what message would you like to send to teachers that might help them to improve the performance of their students on the exam?

Students should know the power series expansions for $\sin x$, $\cos x$ and other functions as specified in the *AP Calculus Course Description*. Students need practice with creating new series from a given series using techniques such as substitution and algebraic manipulation. In using series to estimate functional values, students need practice with error bounds.

The topics listed in the *AP Calculus Course Description* for polynomial approximations and series are an important part of the Calculus BC course. Teachers should devote sufficient time to cover series topics in their courses because they provide the most challenging content of the BC course.

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