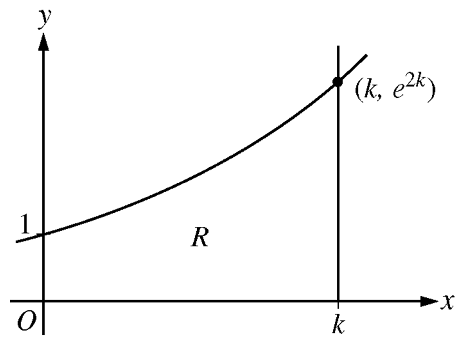
AP® CALCULUS BC
2011 SCORING GUIDELINES

Question 3



Let $f(x) = e^{2x}$. Let R be the region in the first quadrant bounded by the graph of f, the coordinate axes, and the vertical line $x = k$, where $k > 0$. The region R is shown in the figure above.

(a) Write, but do not evaluate, an expression involving an integral that gives the perimeter of R in terms of k.

(b) The region R is rotated about the x-axis to form a solid. Find the volume, V, of the solid in terms of k.

(c) The volume V, found in part (b), changes as k changes. If $\frac{dk}{dt} = \frac{1}{3}$, determine $\frac{dV}{dt}$ when $k = \frac{1}{2}$.

(a) $f'(x) = 2e^{2x}$

Perimeter $= 1 + k + e^{2k} + \int_0^k \sqrt{1 + \left(2e^{2x}\right)^2}\, dx$

$3 : \begin{cases} 1 : f'(x) \\ 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_0^k \left(e^{2x}\right)^2 dx = \pi \int_0^k e^{4x}\, dx = \frac{\pi}{4} e^{4x} \Big|_{x=0}^{x=k} = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$

$4 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

(c) $\frac{dV}{dt} = \pi e^{4k} \frac{dk}{dt}$

When $k = \frac{1}{2}$, $\frac{dV}{dt} = \pi e^2 \cdot \frac{1}{3}$.

$2 : \begin{cases} 1 : \text{applies chain rule} \\ 1 : \text{answer} \end{cases}$

 3A,

NO CALCULATOR ALLOWED

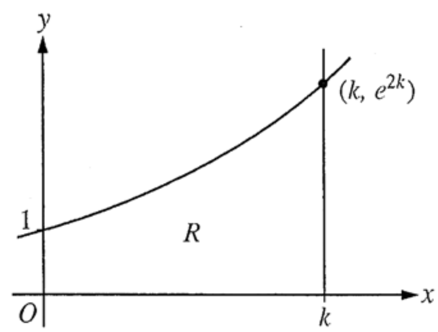
CALCULUS BC

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.



Work for problem 3(a)

$$\frac{dy}{dx} = 2e^{2x}$$

$$P = 1 + k + e^{2k} + \int_0^k \sqrt{1 + (2e^{2x})^2}\, dx$$

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Continue problem 3 on page 9

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Work for problem 3(b)

$u = 4x$

$\frac{du}{dx} = 4$

$$V = \int_0^k \pi r^2 \, dx = \int_0^k \pi \left(e^{2x}\right)^2 dx = \pi \int_0^k e^{4x} \, dx$$

$$= \pi \int_0^k \frac{1}{4} e^u \cdot du = \frac{\pi}{4}\left[e^{4x}\right]_0^k = \frac{\pi}{4}\left(e^{4k} - e^{4(0)}\right)$$

$$= \frac{\pi}{4} e^{4k} - 1\left(\frac{\pi}{4}\right)$$

$$\boxed{V = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}}$$

Work for problem 3(c)

$$\frac{dV}{dt} = \frac{dV}{dk} \cdot \frac{dk}{dt} = \frac{d}{dk}\left(\frac{\pi}{4} e^{4k} - \frac{\pi}{4}\right) \cdot \left(\frac{1}{3}\right) = \frac{\pi}{4}(4) e^{4k} \cdot \left(\frac{1}{3}\right)$$

$$\frac{dV}{dt} = \frac{\pi}{3} e^{4k} \Big|_{k=\frac{1}{2}} = \boxed{\frac{\pi}{3} \cdot e^2}$$

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CALCULUS BC

SECTION II, Part B

Time—60 minutes

Number of problems—4

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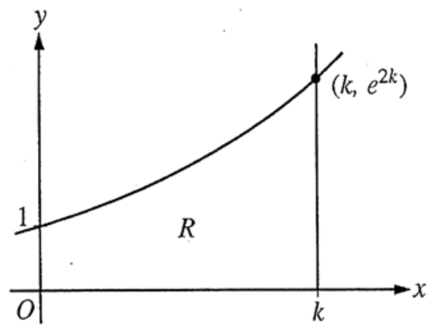

Work for problem 3(a)

$$\int_0^K \sqrt{1+[f'(x)]^2}\,dx = \int_0^K \sqrt{1+[2e^{2x}]^2}\,dx$$

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Continue problem 3 on page 9.

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Work for problem 3(b)

$$V = \pi \int_a^3 f(x)^2 \, dx$$

$$V = \pi \int_0^k (e^{2x})^2 \, dx$$

$$V = \pi \int_0^k e^{4x} \, dx$$

$$V = \pi \frac{1}{4} e^{4x} \Big]_0^k = \boxed{\frac{\pi}{4} e^{4k} - \frac{\pi}{4}}$$

Work for problem 3(c)

$$V = \frac{\pi}{4} e^{4k} - \frac{\pi}{4}$$

$$\frac{dv}{dt} = \pi e^{4k}$$

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3C1

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CALCULUS BC

SECTION II, Part B

Time—60 minutes

Number of problems—4

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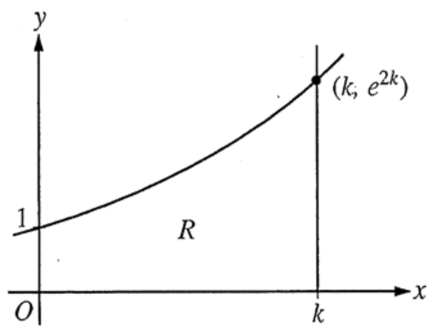

Work for problem 3(a)

Perimeter = R' when $R = \int_0^k e^{2x}$

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Continue problem 3 on page 9.

Work for problem 3(b)

$V = \int_0^k (e^{2x})^2\,dx$

$V = \frac{1}{3}(e^{2x})^3 \cdot \frac{1}{2} \Big|_0^k$

$V = \frac{1}{6}(e^{2x})^3 \Big|_0^k$

$\boxed{V = \frac{1}{6}e^{6k} - \frac{1}{6}}$

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Work for problem 3(c)

$\frac{dV}{dt} = \frac{1}{\cancel{6}} e^{6k} \cdot \cancel{6}\, \frac{dk}{dt}$

$\frac{dV}{dt} = e^{6(1/2)} \left(\frac{1}{3}\right)$

$\boxed{\frac{dV}{dt} = \frac{1}{3}e^3}$

Question 3

Overview

This problem defined a region R in the first quadrant bounded by the graph of $f(x) = e^{2x}$, the coordinate axes, and the line $x = k$, where $k > 0$. A figure depicting R was provided. Part (a) asked for an expression involving an integral that gives the perimeter of R in terms of k. This is the sum of the lengths of the three straight-segment sides and a definite integral giving the length of the curve $y = f(x)$ from $x = 0$ to $x = k$. Part (b) asked for the volume V, in terms of k, of the solid that results from rotating region R about the x-axis. Students needed to set up and evaluate an integral involving the parameter k. Part (c) required students to use the chain rule to find $\frac{dV}{dt}$ when $k = \frac{1}{2}$, given that $\frac{dk}{dt} = \frac{1}{3}$.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), 4 points in part (b), and no points in part (c). In part (a) the student correctly differentiates e^{2x} and presents a correct integral expression for arc length, so the first 2 points were earned. The student's expression for the perimeter is missing terms. In part (b) the student's work is correct. In part (c) the student does not apply the chain rule nor take the derivative of k with respect to t.

Sample: 3C
Score: 4

The student earned 4 points: no points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) the student's work is incorrect. In part (b) the student presents an integral for volume with the correct integrand and correct limits of integration, so the first 2 points were earned. The student incorrectly antidifferentiates $\left(e^{2x}\right)^2$. The student does not include π in the expression for volume. In part (c) the student correctly uses the chain rule to find $\frac{dV}{dt}$ for the volume expression from part (b) and correctly evaluates $\frac{dV}{dt}$ at $k = \frac{1}{2}$, so both points were earned.