AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

Solution	Scoring
(a) $H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2} = \frac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute	1 : answer
(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes. $\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2 \cdot \frac{66+60}{2} + 3 \cdot \frac{60+52}{2} + 4 \cdot \frac{52+44}{2} + 1 \cdot \frac{44+43}{2}\right) = 52.95$	3 : { 1 : meaning of expression 1 : trapezoidal sum 1 : estimate
(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$ The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.	2 : { 1 : value of integral 1 : meaning of expression
(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275$; $\quad H(10) - B(10) = 8.817$ The biscuits are 8.817 degrees Celsius cooler than the tea.	3 : { 1 : integrand 1 : uses $B(0) = 100$ 1 : answer

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2A1

Work for problem 2(a)

$$\frac{H(5) - H(2)}{5-2} = \boxed{\frac{-8}{3} \frac{°C}{min}}$$

Work for problem 2(b)

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{\left[2\left(\frac{66+60}{2}\right) + 3\left(\frac{52+60}{2}\right) + 4\left(\frac{44+52}{2}\right) + 1\left(\frac{43+44}{2}\right)\right]}{10} = \boxed{52.95}$$

THIS ~~[illegible]~~ REPRESENTS THE AVERAGE TEMPERATURE IN DEGREES CELSIUS OF THE TEA ~~[illegible]~~ OVER THE INTERVAL $0 \leq t \leq 10$

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Continue problem 2 on page 7.

2A2

Work for problem 2(c)

$$\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = \boxed{-23°C}$$

THIS EXPRESSION SHOWS THE TOTAL CHANGE IN TEMPERATURE IN DEGREES CELSIUS FROM t=0 to t=10.

Work for problem 2(d)

$$B'(t) = -13.84e^{-.175t}$$

$$B(10) = \int_0^{10} -13.84e^{-.173t} + 100 = 100 - 65.817 = 34.1827$$

$$43 - 34.1827 = 8.817$$

$$= \boxed{8.817°C \text{ cooler}}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

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2B1

Work for problem 2(a)

$H'(3.5) = \frac{H(5) - H(2)}{5-2} = \frac{52-60}{3} = \boxed{\frac{-8}{3}\ {}^{\circ}C/\text{min}}$

Work for problem 2(b)

$\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average value that $H(t)$ is decreasing per min on the interval $0 \le t \le 10$

$\frac{1}{10}\left(f(0) + 2f(2) + 2f(5) + 2f(9) + f(10)\right)$

$\frac{1}{10}\left(66 + 2(60) + 2(52) + 2(44) + 43\right)$

$\frac{1}{10}\left(66 + 120 + 104 + 88 + 43\right)$

$\frac{1}{10}(421) = \boxed{42.1 \approx \frac{1}{10}\int_0^{10} H(t)\,dt}$

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Continue problem 2 on page 7.

2B₂

Work for problem 2(c)

$\int_0^{10} H'(t)\,dt = H(t)\Big]_0^{10} = H(10) - H(0)$

$43 - 66 = \boxed{-23^\circ C}$

This is the total change in temperature of the tea pot in the interval $0 \le t \le 10$

Work for problem 2(d)

$\int_0^{10} B'(t)\,dt = -65.8172$

$100 - 65.8172 = 34.1827$

$43 - 34.1827 = \boxed{8.8172}$

The biscuits are 8.8172°C cooler than the tea.

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t (minutes)	0	2	5	9	10
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2C1

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Work for problem 2(a)

$\frac{52-60}{5-2} = -8/3$ °C/min

Work for problem 2(b)

$\frac{1}{10}\int_0^{10} H(t)$ is the average rate of change in temperature for 0 to 10 minutes.

$\frac{1}{2}h(b_1+b_2)$

$\frac{1}{10}\left(\frac{1}{2}(2)(66+60)+\frac{1}{2}(3)(60+52)+\frac{1}{2}(4)(52+44)+\frac{1}{2}(1)(44+43)\right)$

$\frac{1}{10}(529.5)$

$\frac{1}{10}\int_0^{10} H(t)\,dt \approx 52.95$

Continue problem 2 on page 7.

2C2

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Work for problem 2(c)

$\int_0^{10} H'(t)\,dt = H(t)$ for $0 \le t \le 10$

$\frac{43-66}{10-0} = -2.3$

This is what the temperature in °C should be decreasing at if it was decreasing at a constant °C/minute.

Work for problem 2(d)

tea at $t=10$ is 43°C

$= \int -13.84e^{-.173t}$

$0 = \int -13.84e^{-.173(0)} + C$

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END OF PART A OF SECTION II

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Question 2

Overview

In this problem students were presented with a table giving Celsius temperatures $H(t)$ of a cooling pot of tea during selected times between $t = 0$ and $t = 10$ minutes. Part (a) asked for an approximation for the rate of change of the tea's temperature at time $t = 3.5.$ Students needed to construct a difference quotient using the temperature values across the smallest time interval containing $t = 3.5$ that is supported by the table. Part (b) asked for an interpretation of $\frac{1}{10}\int_0^{10} H(t)\,dt$ and a numeric approximation to this expression using a trapezoidal sum with the four intervals indicated by the table. Students should have recognized this expression as providing the average temperature of the tea, in degrees Celsius, across the time interval $0 \le t \le 10$ minutes. Part (c) asked for an evaluation of and interpretation of $\int_0^{10} H'(t)\,dt.$ Students needed to apply the Fundamental Theorem of Calculus and use values from the table to compute $H(10) - H(0).$ In part (d) students were told about biscuits that were removed from an oven at time $t = 0.$ It is given that the biscuits' temperature was $100°\text{C}$ initially, and that a function $B(t)$ modeling the temperature of the biscuits has derivative $B'(t) = -13.84e^{-0.173t}.$ Students were asked how much cooler the biscuits are than the tea at time $t = 10$ minutes. This was answered by taking the difference between the tea's temperature, $H(10),$ as supplied by the table, and the biscuits' temperature, $B(10),$ computed by $B(10) = 100 + \int_0^{10} B'(t)\,dt.$

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 1 point in part (a), no points in part (b), 2 points in part (c), and 3 points in part (d). In parts (a), (c), and (d) the student's work is correct. Units are not required in part (a) but are required in part (c). In part (b) the student's explanation is inadequate. The student appears to be using the Trapezoidal Rule rather than a general trapezoidal sum.

Sample: 2C
Score: 4

The student earned 4 points: 1 point in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student refers to "the average rate of change in temperature" rather than the average temperature. The student presents a correct trapezoidal sum and estimate, so the second and third points were earned in part (b). In part (c) the student's value is incorrect and the explanation contains no reference to the time interval. In part (d) the student's integral earned the first point.