AP® CALCULUS AB
2011 SCORING GUIDELINES

Question 1

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

(a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

(b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

(c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

(d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

(a) $v(5.5) = -0.45337,\ a(5.5) = -1.35851$

The speed is increasing at time $t = 5.5$, because velocity and acceleration have the same sign.

2 : conclusion with reason

(b) Average velocity $= \frac{1}{6}\int_0^6 v(t)\,dt = 1.949$

2 : { 1 : integral; 1 : answer }

(c) Distance $= \int_0^6 |v(t)|\,dt = 12.573$

2 : { 1 : integral; 1 : answer }

(d) $v(t) = 0$ when $t = 5.19552$. Let $b = 5.19552$.

$v(t)$ changes sign from positive to negative at time $t = b$.

$x(b) = 2 + \int_0^b v(t)\,dt = 14.134$ or 14.135

3 : { 1 : considers $v(t) = 0$; 1 : integral; 1 : answer }

1A,

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

The speed is increasing at $t=5.5$ because $v(5.5)<0$ and $a(5.5)<0$

$v(5.5)=-0.453$

$a(5.5)=-1.358$

Work for problem 1(b)

$$\frac{1}{6-0}\int_0^6 v(t)\,dt = 1.949$$

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Continue problem 1 on page 5.

1 A_2

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Work for problem 1(c)

$\int_0^6 |v(t)|\,dt = 12.573$

Work for problem 1(d)

$v(t) = 0$ at $t = 5.1955223$ and $v(t)$ changes signs at $t = 5.1955223$

$X(5.1955223) = 2 + \int_0^{5.195523} v(t)\,dt$

position at $t = 5.1955223$ is 14.134

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1B,

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SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

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Work for problem 1(a)

a) The speed of the particle at $t=5.5$ in increasing as both $v(t)$ and $a(t)$ are negative, so $v(t)$ is getting faster in the negative direction. In other words

speed $= |v(t)|$

$|v(t)|$ is increasing

Work for problem 1(b)

$$\frac{v(6)+v(0)}{6-0} = .289$$

Continue problem 1 on page

Work for problem 1(c)

$v(t) = 2\sin(e^{t/4}) + 1$

1B2

$\int_0^6 |v(t)|\,dt = 12.573$

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Work for problem 1(d)

$x(5.196) = \int_0^{5.196} (v(t)) + 2 = 14.135$

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1C₁

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

At time $t=5.5$, the speed of the particle is increasing, because the acceleration and velosity of the particle at time $t=5.5$ are both negative,

Work for problem 1(b)

Average Velocity $= \frac{v(6)-v(0)}{6-0} \approx \boxed{-0.453 \ {}^{u}/_{t}}$

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Continue problem 1 on page

1C2

Work for problem 1(c)

Total Distance Traveled = $2 + \int_0^6 |v(t)|\,dt = 2 + 12.573 = \boxed{14.573\text{ u}}$

We must take the absolute value of v(t) to find the total distance traveled.

Work for problem 1(d)

The particle changes direction when velocity, v(t), changes sign. On the interval (0, 6), velocity changes sign from positive to negative once at time t = 5.196. Therefore, the particle changes its direction once at time t = 5.196.

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AP® CALCULUS AB
2011 SCORING COMMENTARY

Question 1

Overview

This problem presented students with a particle in rectilinear motion during the time interval $0 \le t \le 6$. The position, $x(t)$, of the particle is unknown, but velocity and acceleration functions, $v(t)$ and $a(t)$, respectively, are provided. Part (a) asked students whether the speed of the particle is increasing or decreasing at time $t = 5.5$. Students should have evaluated both the velocity and the acceleration functions at $t = 5.5$; because $v(5.5) < 0$ and $a(5.5) < 0$, the particle's speed is increasing. Part (b) asked for the average velocity of the particle during the given time interval. This can be computed as an average value, $\frac{1}{6-0}\int_0^6 v(t)\,dt$, and evaluated on a calculator. Part (c) asked for the total distance traveled by the particle. The total distance is the value of $\int_0^6 |v(t)|\,dt$, which can be computed directly on the calculator, or by splitting the interval into a segment on which $v(t) > 0$ and one on which $v(t) < 0$, and then appropriately combining the corresponding definite integrals of velocity. Part (d) highlighted that the particle changes direction exactly once during the interval, and asked for the position of the particle at that time. If they had not already done so, students should have used their calculators to find the solution to $v(t) = 0$ with $0 \le t \le 6$. If the solution is $t = A$, the position of the particle at that time is then calculated as $x(A) = 2 + \int_0^A v(t)\,dt$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). In parts (a) and (c) the student's work is correct. In part (b) the student's work is incorrect. In part (d) the student does not write $v(t) = 0$, so the first point was not earned. The student earned the integral and answer points.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student finds average acceleration, so no points were earned. In part (c) the student earned 1 point for the integral. Because the student incorrectly adds 2, the answer point was not earned. In part (d) the student's phrase "velocity, $v(t)$, changes sign" earned the first point. The student finds the value $t = 5.196$ but does not continue.