AP® CALCULUS BC
2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813 \text{ mm}$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{S(60) - S(0)}{60} = 2.863 \text{ or } 2.864 \text{ mm/day}$

1 : answer

(c) $V(t) = 100\pi S(t)$
$V'(7) = 100\pi S'(7) = 602.218$

The volume of water in the can is increasing at a rate of $602.218 \text{ mm}^3/\text{day}$.

$2 : \begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

$2 : \begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

1 : units in (b) or (c)

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

1(a). $\int_0^{60} 2\sin(0.03t)+1.5\, dt$

≈ 171.813 mm

Work for problem 1(b)

$S(60) = \int_0^{60} 2\sin(0.03t)+1.5\, dt$

$= 171.83$ mm.

$\frac{S(60)-S(0)}{60 \text{ days}}$ average rate of change in height.

$\frac{171.813 - 0 \text{ mm}}{60 \text{ days}} \approx 2.864$ mm/day

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Continue problem 1 on page 5.

Work for problem 1(c)

$V=\pi r^2 h$

$\frac{dV}{dt} = \pi\cdot 100\cdot\frac{dh}{dt}$

At $t=7$ $\frac{dh}{dt} = 2\cdot\sin(0.03\cdot 7)+1.5$

≈ 1.917

$\frac{dV}{dt} = \pi\cdot 100\cdot 1.917$

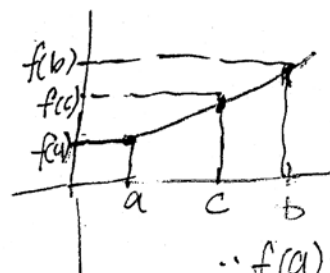
$\frac{dV}{dt} \approx 602.218 \text{ mm}^3/\text{day}$

Work for problem 1(d)

$M(t) = \frac{3}{400}t^3 - \frac{3}{40}t^2 + \frac{33}{40}t$

$M'(t) = 3\cdot\frac{3}{400}t^2 - 2\cdot\frac{3}{40}t + \frac{33}{40}$

$= \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40}$



$D(t) = \frac{9}{400}t^2 - \frac{3}{20}t + \frac{33}{40} - 2\sin(0.03t)+1.5$

$D(0) = \frac{33}{40} - 1.5 = -0.675.$

$D(60) = 69.377$

$\because f(a) = -0.675 < 0$

$f(b) = 69.377 > 0$

$\therefore$ There exists a time c which $f(c)$ or $D=0$ $(M'(t)-S'(t)=0)$

In order for both the cans' heights to change at the same rate $D(t)=0 \rightarrow M'(t)-S'(t)=0$.

$\therefore$ According to the IVT. If a function is continuous on the interval $[a,b]$, and there exist corresponding value $f(a)$ & $f(b)$. in which $f(a) < f(b)$, the c, a value in between (a,b) on the interval $[a,b]$. has a corresponding value in between $f(a)$ & $f(b)$.

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CALCULUS BC

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

$$\int_0^{60} S'(t)\,dt$$

$$= \int_0^{60} 2\sin(0.03t)+1.5\,dt$$

$= 171.813$ millimeters

Work for problem 1(b)

$$\frac{1}{60-0}\int_0^{60} S'(t)\,dt$$

$$= \frac{1}{60}\int_0^{60} 2\sin(0.03t)+1.5\,dt$$

$= 2.86356$ mill/day

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Continue problem 1 on page 5.

Work for problem 1(c)

$V = \pi r^2 h = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$= 100\pi(2\sin(0.03 \cdot 7) + 1.5)$

$= 602$ mil^3/day at $t=7$

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Work for problem 1(d)

$M'(t) = \frac{1}{400}(9t^2 - 60t + 330)$

$D(t) = \left(\frac{1}{400}(9t^2 - 60t + 330)\right) - \quad 2\sin(0.03t) + 1.5$

$M'(t) = S'(t)$ at $t = 11.8166$

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GO ON TO THE NEXT PAGE.

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

Work for problem 1(a)

$\int_0^{60} S'(t)\,dt = \int_0^{60} (2\sin(0.03t)+1.5)\,dt = 171.183$ millimeters

Work for problem 1(b)

$S(t) = \int S'(t)\,dt = 1.5x - 66.67\cos(0.03t)+c$

$S(0)=0$

$c = 66.67$, $0 = 1.5(0) - 66.67\cos(0.03(0)) + c$

$S(t) = 1.5x - 66.67\cos(0.03t) + 66.67$

average rate of change =

$\frac{S(60)-S(0)}{60} = \frac{171.818 - 0}{60} = 28.636$ mm/day

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Continue problem 1 on page 5

Work for problem 1(c)

$V = \pi r^2 h$

$V = \pi (10)^2 h$

$V = 100\pi h$

$\frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$S'(7) = \frac{dh}{dt}\Big|_{t=7}$

$S'(7) = 1.917$ mm/day

$\frac{dV}{dt}\Big|_{t=7} = 100\pi(1.917)$

$= 602.243$ mm^3/day

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Work for problem 1(d)

$M'(t) = \frac{9x^2}{400} - \frac{3x}{20} + \frac{33}{40}$

$\int_0^{60} D(t)\,dt = \int_0^{60} M'(t) - S'(t)\,dt$

$\int_0^{60} D(t)\,dt = (60-0)\,D(c)$

$D(c) = 20.4614$

GO ON TO THE NEXT PAGE.

Question 1

Sample: 1A
Score: 9

The student earned all 9 points. In part (d) the student considers $D(0)$ and $D(60)$, notes that they have opposite signs, implies that D is continuous, and invokes the Intermediate Value Theorem to conclude that $D(t)$ must equal 0 for some t in the interval.

Sample: 1B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 1 point in part (c), no points in part (d), and the units point. In parts (a) and (b) the student's work is correct. In part (c) the student earned the first point with the substitution for $S'(7)$ in the expression for $\frac{dV}{dt}$. Prior to that step, the student was working with $\frac{dh}{dt}$ rather than $S'(t)$. The student's answer is not presented accurately to three decimal places. In part (d) the student's work is incorrect.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no point in part (b), 1 point in part (c), no points in part (d), and the units point. In part (a) the student has the correct limits and integrand but presents an incorrect answer of 171.183 and so earned 2 of the 3 points. In part (b) the student's decimal point is incorrectly placed. In part (c) the student establishes the relationship between V and S by connecting $\frac{dV}{dt}$ to $\frac{dh}{dt}$ and $\frac{dh}{dt}$ to S'. The student uses the truncated value 1.917 for $S'(7)$ in the computation of $\frac{dV}{dt}$, so the student's answer is incorrect. In part (d) the student's work is incorrect.