General Notes

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for the solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong — for example, a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point and a student's solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exams equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams and what is expected for each, see "The Free-Response Sections — Student Presentation" in the *AP Physics Course Description*.

4. The scoring guidelines typically show numerical results using the value $g = 9.8\ \text{m/s}^2$, but use of $10\ \text{m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 3

15 points total | **Distribution of points**

(a) 4 points

For a correct relationship between velocity and acceleration — 1 point

$$\upsilon = \int a(t)\,dt \quad \textbf{OR} \quad \upsilon = \int_0^t a(t)\,dt \quad \textbf{OR} \quad \frac{d\upsilon}{dt} = a$$

For a correct substitution of the expression for acceleration into the integral relationship — 1 point

$$\upsilon = \int\left(a_{\max}\sin\frac{\pi t}{T}\right)dt \quad \textbf{OR} \quad \upsilon = \int_0^t\left(a_{\max}\sin\frac{\pi t}{T}\right)dt \qquad (0 < t < T)$$

For a correct evaluation of the integral, with an integration constant or correct limits — 1 point

$$\upsilon = -\frac{a_{\max}T}{\pi}\cos\frac{\pi t}{T} + C \quad \textbf{OR} \quad \upsilon = -\frac{a_{\max}T}{\pi}\cos\frac{\pi t}{T}\Bigg|_0^t \qquad (0 < t < T)$$

For a correct determination of the integration constant or evaluation between the limits — 1 point

$$\upsilon(0) = -\frac{a_{\max}T}{\pi} + C = 0 \;\Rightarrow\; C = \frac{a_{\max}T}{\pi} \quad \textbf{OR} \quad \upsilon = -\frac{a_{\max}T}{\pi}\left(\cos\frac{\pi t}{T} - 1\right) \qquad (0 < t < T)$$

$$\upsilon = \frac{a_{\max}T}{\pi}\left(1 - \cos\frac{\pi t}{T}\right) \qquad (0 < t < T)$$

(b) 2 points

For indicating that the work done by the net force is equal to the change in kinetic energy — 1 point

$$W = \frac{1}{2}m\left(\upsilon_f^2 - \upsilon_i^2\right)$$

For a correct substitution of velocity from (a) into the work-energy expression — 1 point

$$\upsilon_f = \upsilon_T = \frac{a_{\max}T}{\pi}(1 - \cos\pi) = \frac{2a_{\max}T}{\pi}$$

$$\upsilon_i = \upsilon_0 = \frac{a_{\max}T}{\pi}(1 - \cos 0) = 0$$

$$W = \frac{1}{2}m\left(\frac{2a_{\max}T}{\pi}\right)^2$$

$$W = \frac{2ma_{\max}^2T^2}{\pi^2}$$

Alternate solution (integral form) — *Alternate points*

$$W = \int F \bullet dx$$

For a correct substitution of the expression for force into the integral — *1 point*

$$W = \int ma_{\max}\sin\frac{\pi t}{T}\,dx$$

For a correct expression for dx in terms of time — *1 point*

Question 3 (continued)

Distribution of points

(b) (continued)

$$W = \int_0^T ma_{\max} \sin\frac{\pi t}{T} \left(\frac{a_{\max}T}{\pi}\left(1-\cos\frac{\pi t}{T}\right)\right)dt$$

$$W = \frac{ma_{\max}^2 T}{\pi}\int_0^T\left(\sin\frac{\pi t}{T} - \left(\sin\frac{\pi t}{T}\cos\frac{\pi t}{T}\right)\right)dt$$

$$W = \frac{ma_{\max}^2 T}{\pi}\int_0^T\left(\sin\frac{\pi t}{T} - \frac{1}{2}\sin\frac{2\pi t}{T}\right)dt$$

$$W = \frac{ma_{\max}^2 T^2}{\pi^2}\left(-\cos\frac{\pi t}{T} - \frac{1}{4}\cos\frac{2\pi t}{T}\right)\Bigg|_0^T$$

$$W = \frac{2ma_{\max}^2 T^2}{\pi^2}$$

(c) 1 point

Starting with Newton's second law:

$F_{net} = F_{rope} - mg\sin\theta = ma$

At terminal velocity, the net force and acceleration are zero:

$F_{rope} - mg\sin\theta = 0$

For a correct expression for the force — 1 point

$F_{rope} = mg\sin\theta$

(d) 2 points

$J = \int F\,dt$

For a correct substitution of force into the impulse-time relationship — 1 point

$$J = ma_{\max}\int_0^T \sin\frac{\pi t}{T}\,dt$$

$$J = \frac{ma_{\max}T}{\pi}\left(-\cos\frac{\pi t}{T}\right)\Bigg|_0^T$$

For evaluation at the limits of integration — 1 point

$$J = \frac{ma_{\max}T}{\pi}\left[-\cos\pi + \cos 0\right]$$

$$J = \frac{2ma_{\max}T}{\pi}$$

Alternate solution (impulse-momentum) — *Alternate points*

$J = \Delta p = m\upsilon_T$

Question 3 (continued)

Distribution of points

(d) (continued)

For a correct substitution of the velocity — *1 point*

$$J = \frac{ma_{\max}T}{\pi}\left(1 - \cos\frac{\pi t}{T}\right)$$

For setting $t = T$ — *1 point*

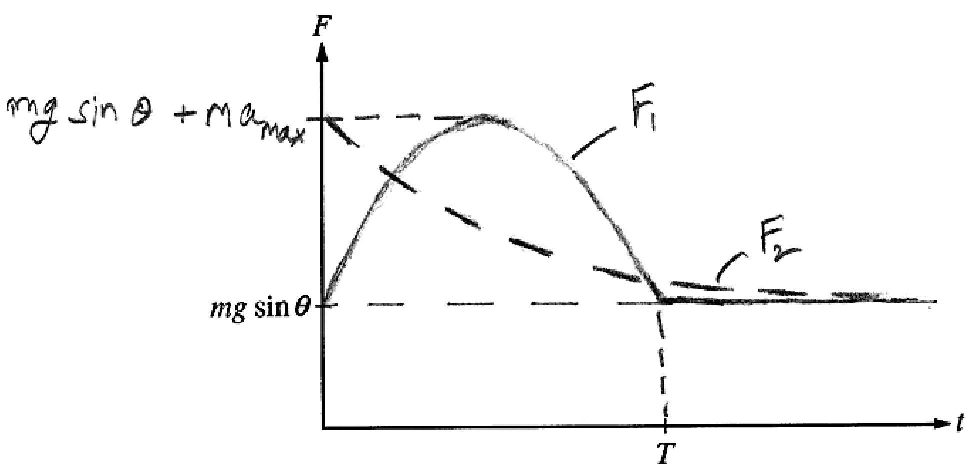
$$J = \frac{ma_{\max}T}{\pi}(1 - \cos\pi)$$

$$J = \frac{2ma_{\max}T}{\pi}$$

(e) 6 points

$$F_1 = mg\sin\theta + ma_{\max}\sin\left(\frac{\pi t}{T}\right) \qquad (0 < t < T)$$

$$F_2 = mg\sin\theta + ma_{\max}\, e^{-\pi t/2T}$$

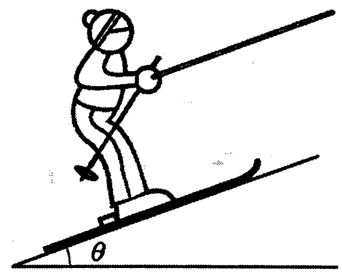


For a graph labeled F_1:

- for starting at $mg\sin\theta$ — 1 point
- for half a sine wave with a maximum at $\sim T/2$ — 1 point
- for returning to original starting point at $t = T$ — 1 point
- for a horizontal line at the original starting point for $t > T$ — 1 point

For a graph labeled F_2:

- for starting on the vertical axis at a point above the starting point of F_1 (if there is no F_1 graph, this point was awarded if the F_2 graph starts above $mg\sin\theta$) — 1 point
- for an exponential decay graph — 1 point



Mech. 3.

A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$
\begin{aligned}
a &= a_{max} \sin\frac{\pi t}{T} \qquad (0 < t < T) \\
&= 0 \qquad\qquad\qquad (t \geq T),
\end{aligned}
$$

where a_{max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

$$v = \int_0^t a\,dt = \int a_{max} \sin\frac{\pi t}{T}\,dt = a_{max}\left[-\frac{T}{\pi}\cos\frac{\pi t}{T}\right]_0^t$$

$$\boxed{-a_{max}\frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) + a_{max}\frac{T}{\pi} \qquad (0<t<T)}$$

$\cancel{\int a\,dt = 0 + c}$

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

$$W = \Delta K = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 = \tfrac{1}{2}\cdot m\left(2a_{max}\frac{T}{\pi}\right)^2 - 0$$

$$= \frac{2 m a_{max}^2 T^2}{\pi^2}$$

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

$\Sigma F_x = 0$

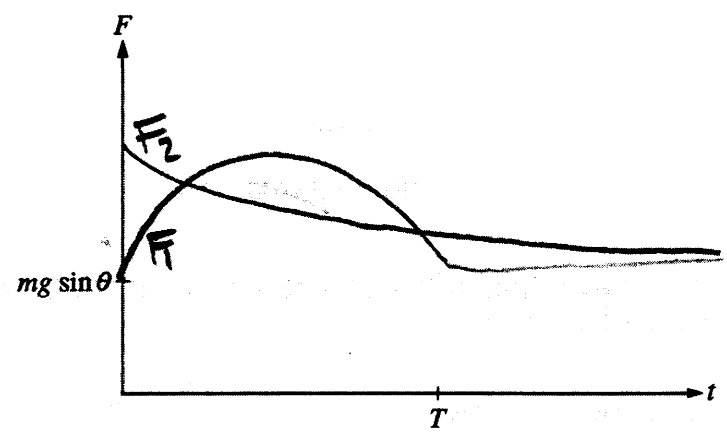
$F_{rope} - mg\sin\theta = 0$

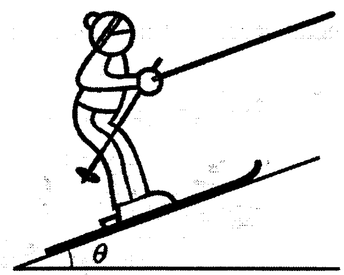
$F_{rope} = mg\sin\theta$

(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

$P = m\Delta v = m\,2a_{max}\frac{T}{\pi}$

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max}e^{-\pi t/2T}$ for all $t > 0$, where a_{max} and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.





Mech. 3.

A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{max} \sin\frac{\pi t}{T} \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T),$$

where a_{max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

$a = \frac{dv}{dt}$

$\int_0^T a \cdot dt = \int_0^v dv$

$\int_0^T a \sin\frac{\pi t}{T} = \int_0^v dv$

$\frac{aT \cos\left(\frac{\pi t}{T}\right)}{\pi} = v \quad 0<t<T$

$v = \frac{aT}{\pi} \quad t \geq T$

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

$W = \int F \cdot dr$

$F = ma$

$F = m\, a_{max} \sin\frac{\pi t}{T}$

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

$F_{net} = ma$ $a = 0$

$\boxed{F_{net} = 0}$

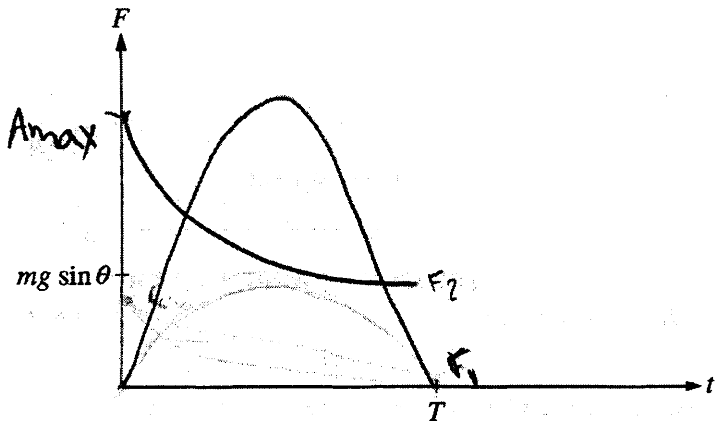
(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

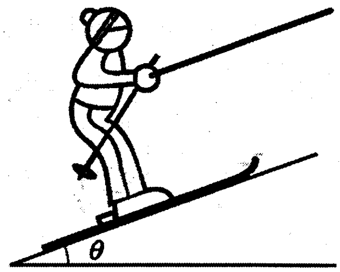
$J = \int_0^T F \cdot dt$

$J = \int_0^T m\, a_{max} \sin \frac{\pi t}{T}$

$\boxed{J = \frac{2 M a_{max} T}{\pi}}$

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max} e^{-\pi t/2T}$ for all $t > 0$, where a_{max} and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.





Mech. 3.

A skier of mass m will be pulled up a hill by a rope, as shown above. The magnitude of the acceleration of the skier as a function of time t can be modeled by the equations

$$a = a_{max} \sin\frac{\pi t}{T} \qquad (0 < t < T)$$
$$= 0 \qquad (t \geq T),$$

where a_{max} and T are constants. The hill is inclined at an angle θ above the horizontal, and friction between the skis and the snow is negligible. Express your answers in terms of given quantities and fundamental constants.

(a) Derive an expression for the velocity of the skier as a function of time during the acceleration. Assume the skier starts from rest.

$$V = \cancel{V_0^2}^{\,0} + 2a\Delta x$$

$$V = 2\left(a_{max} \sin\frac{\pi t}{T}\right)\Delta x$$

(b) Derive an expression for the work done by the net force on the skier from rest until terminal speed is reached.

$$W = \int_0^{v_t} (F)\, dx$$

$$W = F\Delta x$$

(c) Determine the magnitude of the force exerted by the rope on the skier at terminal speed.

f of rope = tension

$f \sin\theta$

$m\left(a_{max} \sin\frac{\pi t}{t}\right)\sin\theta$ N

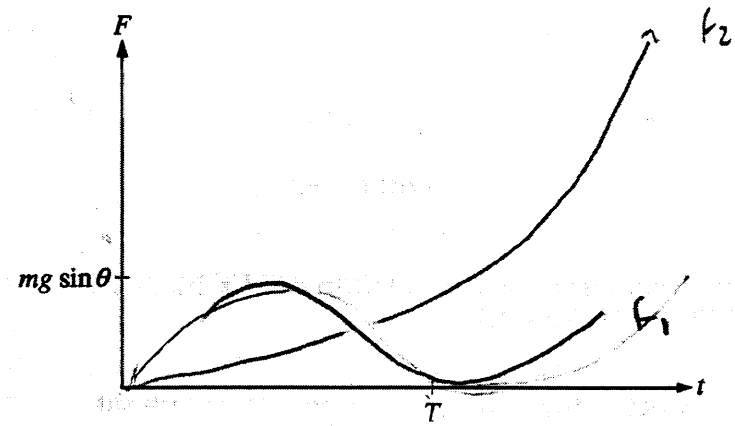
(d) Derive an expression for the total impulse imparted to the skier during the acceleration.

(kg m/s²) m   kg m

$J = F\Delta x$

$J = m\left(a_{max} \sin\frac{\pi t}{T}\right)\Delta x$ kg m/s²

(e) Suppose that the magnitude of the acceleration is instead modeled as $a = a_{max}e^{-\pi t/2T}$ for all $t > 0$, where a_{max} and T are the same as in the original model. On the axes below, sketch the graphs of the force exerted by the rope on the skier for the two models, from $t = 0$ to a time $t > T$. Label the original model F_1 and the new model F_2.



$F = ma$

$F = m\left(a_{max}e^{-\pi t/2t}\right)$

constant

$F = m\left(a_{max}\sin\frac{\pi t}{T}\right)$

Question 3

Overview

The intent of this question was to have students look at a mechanics problem involving a segmented acceleration that varied with time, both conceptually and computationally. The problem involved extensive use of calculus and required conceptual knowledge in order to draw two graphs.

Sample: M3-A
Score: 15

This full-credit response has a minimal amount of work shown. This can be detrimental to earning partial credit in some cases, if it is impossible to determine whether the intermediate work is correct.

Sample: M3-B
Score: 8

Part (a) earned 2 points for the integral expression for velocity and substitution of the expression for acceleration into the integral. Note that the integration is considered incorrect because there is no constant of integration or limits on the integration product. Parts (b) and (c) earned no credit. Part (d) earned full credit. Evaluation at the correct limits of integration is implied by the correct answer. Part (e) earned 4 points because the F_1 graph does not start at $mg\sin\theta$ and does not continue past $t = T$.

Sample: M3-C
Score: 2

Parts (a) through (d) earned no credit. Part (e) earned 2 points for the F_1 graph describing half of a sine wave peaking at time $T/2$ and returning to its original starting point at time T.