General Notes

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for the solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong — for example, a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth 1 point and a student’s solution contains the application of that equation to the problem but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exams equation sheet. For a description of the use of such terms as “derive” and “calculate” on the exams and what is expected for each, see “The Free-Response Sections — Student Presentation” in the AP Physics Course Description.

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of $10 \text{ m/s}^2$ is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.
Question 5

10 points total

(a) 2 points

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]
\[ \sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} \]

For correct substitutions into Snell’s Law 1 point

\[ \sin \theta_2 = \frac{(1.0) \sin 40^\circ}{1.65} = 0.390 \]

For the correct answer 1 point

\[ \theta_2 = 22.9^\circ \text{ or } 23^\circ \]

(b) 3 points

In order for total internal reflection to occur, \( \theta_3 \) must increase until it is greater than \( \theta_{\text{critical}} \). For this to occur, \( \theta_2 \) must decrease. Finally, to decrease \( \theta_2 \) there must be a decrease in \( \theta_1 \).

For stating that \( \theta_3 \) must increase to become greater than \( \theta_{\text{critical}} \) 1 point

For stating that \( \theta_2 \) must decrease 1 point

For stating that \( \theta_1 \) must decrease 1 point

Alternate solution

Alternate points

For calculating the minimum value of \( \theta_3 \) that will result in total internal reflection 1 point

\[ \theta_{\text{critical}} = \sin^{-1} \left( \frac{1}{1.65} \right) = 37.3^\circ \]

For calculating the corresponding value of \( \theta_2 \) 1 point

\[ \theta_2 = 60^\circ - \theta_3 = 22.7^\circ \]

For calculating the corresponding value of \( \theta_1 \) 1 point

\[ \theta_1 = \sin^{-1} [1.65(\sin 22.7^\circ)] = 39.5^\circ, \text{ therefore } \theta_1 \text{ must be decreased.} \]

Other correct methods also received appropriate credit.
(c) 2 points

For a correct relationship between the wavelength in air and the wavelength in the film, which can be derived from $\lambda = \nu f$ and $n = c/\nu$

$$\lambda_{\text{film}} = \frac{\lambda_{\text{air}}}{n_{\text{film}}}$$

$$\lambda_{\text{film}} = \frac{6.65 \times 10^{-7} \text{ m}}{1.38}$$

For the correct answer with units 1 point

$$\lambda_{\text{film}} = 4.82 \times 10^{-7} \text{ m}$$

Alternate solution

Alternate points

For the correct calculation of either the velocity of light in the medium or the frequency of the light 1 point

$$v_{\text{film}} = \frac{c}{n_{\text{film}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.38} = 2.17 \times 10^8 \text{ m/s}$$

OR $$f = \frac{c}{\lambda_{\text{air}}} = \frac{3.00 \times 10^8 \text{ m/s}}{6.65 \times 10^{-7} \text{ m}} = 4.51 \times 10^{14} \text{ Hz}$$

$$\lambda_{\text{film}} = \frac{v_{\text{film}}}{f} = \frac{2.17 \times 10^8 \text{ m/s}}{4.51 \times 10^{14} \text{ Hz}}$$

For the correct answer with units 1 point

$$\lambda_{\text{film}} = 4.81 \times 10^{-7} \text{ m}$$

(ii) 3 points

The light that enters the film and reflects off the prism travels a total distance $2t$ through the film. At both interfaces, there is a 180° phase change when the light is reflected, so the relative phase change of the interfering rays is zero. For destructive interference, the minimum path length in the film must equal $\lambda_{\text{film}}/2$. Therefore, we have the relationship $2t = \lambda_{\text{film}}/2$.

For the relationship between the thickness of the film and the wavelength of light in the film 1 point

$$t = \frac{\lambda_{\text{film}}}{4} \text{ or } t = \frac{\lambda_{\text{air}}}{4n_{\text{film}}}$$

For the correct substitution of the wavelength of light in the film 1 point

$$t = \frac{\lambda_{\text{film}}}{4} = \frac{4.82 \times 10^{-7} \text{ m}}{4} \text{ OR } t = \frac{\lambda_{\text{air}}}{4n_{\text{film}}} = \frac{6.65 \times 10^{-7} \text{ m}}{4(1.38)}$$

For the correct answer with units 1 point

$$t = 1.20 \times 10^{-7} \text{ m}$$
5. (10 points)

As shown above, a beam of red light of wavelength \(6.65 \times 10^{-7}\) m in air is incident on a glass prism at an angle \(\theta_1\) with the normal. The glass has index of refraction \(n = 1.65\) for the red light. When \(\theta_1 = 40^\circ\), the beam emerges on the other side of the prism at an angle \(\theta_4 = 84^\circ\).

(a) Calculate the angle of refraction \(\theta_2\) at the left side of the prism.

\[
\frac{n_1 \sin \theta_1}{n_2 \sin \theta_2} = \frac{1.00 \sin 40^\circ}{1.65 \sin \theta_2}
\]

\(\theta_2 = 23^\circ\)

(b) Using the same prism, describe a change to the setup that would result in total internal reflection of the beam at the right side of the prism. Justify your answer.

\(\theta_3 = 60 - \theta_2\)

\[
\frac{n_1 \sin \theta_3}{n_2 \sin \theta_c} = \frac{1.65 \sin 39.3^\circ}{1}
\]

\(\theta_c = 39.3^\circ\)

\(\theta_2\) have to be smaller than 22.7°. So \(\theta_1\) have to be smaller than 39.5° using Snell's law.

(c) The incident beam is now perpendicular to the surface. The glass is coated with a thin film that has an index of refraction \(n_f = 1.38\) to reduce the partial reflection of the beam at this angle.

i. Calculate the wavelength of the red light in the film.

\[
\lambda_2 = \frac{\lambda_1}{n_2} = \frac{6.65 \times 10^{-7} \text{ m}}{1.65} = 4.03 \times 10^{-7} \text{ m}
\]

ii. Calculate the minimum thickness of the film for which the intensity of the reflected red ray is near zero.

air-film interface: \(\frac{1}{2} \lambda\)

film-glass interface: \(2t + \frac{1}{2} \lambda\)

total interference: \(2t + \frac{1}{2} \lambda - \frac{1}{2} \lambda = (m + \frac{1}{2}) \lambda\) (destructive interference)

\(2t = (m + \frac{1}{2}) \lambda\)

minimum thickness when \(m = 0\)

\[
t = \frac{1}{2} (4.03 \times 10^{-7}) \text{ m} = 1.01 \times 10^{-7} \text{ m}
\]

Go on to the next page.
5. (10 points)

As shown above, a beam of red light of wavelength $6.65 \times 10^{-7}$ m in air is incident on a glass prism at an angle $\theta_1$ with the normal. The glass has index of refraction $n = 1.65$ for the red light. When $\theta_1 = 40^\circ$, the beam emerges on the other side of the prism at an angle $\theta_4 = 84^\circ$.

(a) Calculate the angle of refraction $\theta_2$ at the left side of the prism.

\[
\sin 40^\circ = 1.65 \sin \theta_2
\]

\[
\theta_2 = 22.928^\circ
\]

(b) Using the same prism, describe a change to the setup that would result in total internal reflection of the beam at the right side of the prism. Justify your answer.

If the incident beam, entered when $\theta_1$ was closer to the normal, it would internally reflect since $\theta_2$ would decrease.

(c) The incident beam is now perpendicular to the surface. The glass is coated with a thin film that has an index of refraction $n_f = 1.38$ to reduce the partial reflection of the beam at this angle.

i. Calculate the wavelength of the red light in the film.

\[
\lambda = \frac{c}{n_f} = \frac{4.151 \times 10^{14}}{1.38} = 4.819 \times 10^{-7} \text{ m}
\]

ii. Calculate the minimum thickness of the film for which the intensity of the reflected red ray is near zero.

\[
d \sin \theta = \frac{\lambda}{n_f} = 4.819 \times 10^{-7}
\]

\[
\lambda = 4.819 \times 10^{-7} \text{ m}
\]
5. (10 points)

As shown above, a beam of red light of wavelength $6.65 \times 10^{-7}$ m in air is incident on a glass prism at an angle $\theta_1$ with the normal. The glass has index of refraction $n = 1.65$ for the red light. When $\theta_1 = 40^\circ$, the beam emerges on the other side of the prism at an angle $\theta_4 = 84^\circ$.

(a) Calculate the angle of refraction $\theta_2$ at the left side of the prism.

$$n \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$$

$$n = 1.0 \cdot (\sin 40^\circ) = 1.65 \cdot (\sin \theta_2)$$

$$\sin \theta_2 = \frac{0.642}{1.65}$$

$$\theta_2 = \sin^{-1}(0.389) = 22.98^\circ$$

(b) Using the same prism, describe a change to the setup that would result in total internal reflection of the beam at the right side of the prism. Justify your answer.

Put the right side of the prism in a medium that has $n > 1.65$. If the third medium has greater $n$ than the second, the angle of reflection will be greater and cause a internal reflection.

(c) The incident beam is now perpendicular to the surface. The glass is coated with a thin film that has an index of refraction $n_f = 1.38$ to reduce the partial reflection of the beam at this angle.

i. Calculate the wavelength of the red light in the film.

$$n = \frac{c}{v}$$

$$1.38 = \frac{3.0 \times 10^8 \text{ m/s}}{v}$$

$$v = \frac{3.0 \times 10^8 \text{ m/s}}{1.38}$$

$$\lambda = \frac{c}{v} = \frac{3.0 \times 10^8 \text{ m/s}}{\frac{1.38}{f}}$$

ii. Calculate the minimum thickness of the film for which the intensity of the reflected red ray is near zero.

$$2t = m(1 + \lambda)$$
Question 5

Overview

This question probed students’ knowledge of refraction, total internal reflection, changes in light waves as they travel between different media, and thin film interference.

Sample: B5-A
Score: 9

In this response the only point lost is in part (c)(i), which has an incorrect value for the index of refraction. Note that in part (b) the student explicitly states the relationship between $\theta_2$ and $\theta_3$. In part (c)(ii) the substitution of the answer from (c)(i) is acceptable and the answer is consistent with that substitution, thus earning the full 3 points. The student derives the necessary relationship from the phase differences.

Sample: B5-B
Score: 6

Part (a) earned full credit. In part (b) 2 points were earned because there are statements about $\theta_1$ and $\theta_2$, but no logical connection to total internal reflection or $\theta_3$ is made. Part (c)(i) received full credit. Part (c)(ii) contains no relevant work, so no points were earned.

Sample: B5-C
Score: 3

Part (a) earned full credit. In part (b) the proposed change would actually reduce the value of $\theta_4$ instead of increasing it to 90°, so no points were earned. The desired effect would occur only if the index of refraction of the added material was smaller than that of air. Part (c)(i) earned 1 point for correctly calculating the speed of light in the film. Part (c)(ii) earned no points.