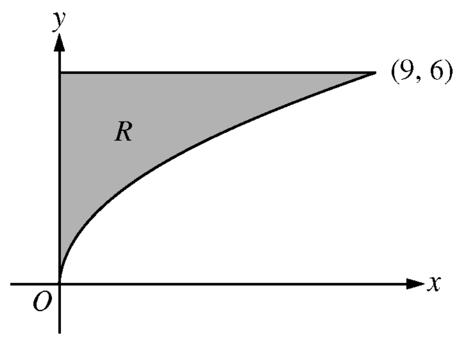
AP® CALCULUS BC
2010 SCORING GUIDELINES

Question 4



Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the y-axis, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.

(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) Area $= \int_0^9 \left(6 - 2\sqrt{x}\right) dx = \left(6x - \frac{4}{3}x^{3/2}\right)\Big\|_{x=0}^{x=9} = 18$	$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi\int_0^9 \left(\left(7 - 2\sqrt{x}\right)^2 - (7 - 6)^2\right) dx$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$
(c) Solving $y = 2\sqrt{x}$ for x yields $x = \frac{y^2}{4}$. Each rectangular cross section has area $\left(3\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right) = \frac{3}{16}y^4$. Volume $= \int_0^6 \frac{3}{16}y^4\, dy$	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

NO CALCULATOR ALLOWED

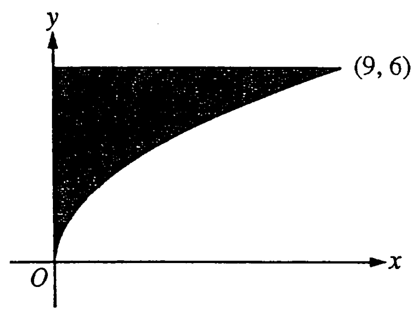
CALCULUS BC

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$R = \int_0^9 6\,dx - \int_0^9 2\sqrt{x}\,dx$

$54 - 2\left[\frac{2x^{3/2}}{3}\Big|_0^9\right]$

$9^{3/2} = 27$

$\frac{27}{3} = 9$

$54 - 2[18 - 0]$

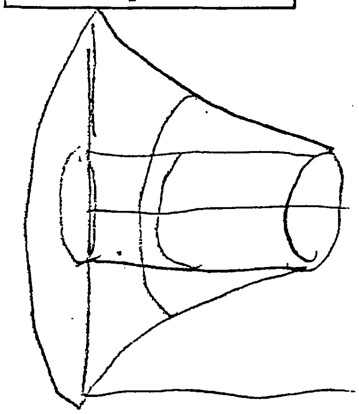
$54 - 36$

$\boxed{\text{Area} = 18}$ units^2

Continue problem 4 on page 1

NO CALCULATOR ALLOWED

Work for problem 4(b)



Vertical Washers $\cancel{\pi(R^2-r^2)}$

dx = thickness

$(\pi R^2 - \pi r^2)\,dx$

$\pi(R^2-r^2)$

$R = 7-2\sqrt{x}$

$r = 7-6$

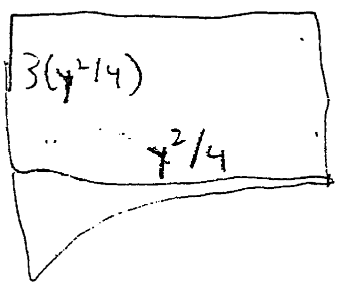
$r=1$

$\boxed{\text{Volume} = \pi\int_0^9 \left[(7-2\sqrt{x})^2 - 1\right]dx}$

Work for problem 4(c)

$2\sqrt{x}=y \quad 4\cdot x = y^2$

$x = \frac{y^2}{4}$



$\int_0^6 \left[\frac{y^2}{4}\cdot\frac{3y^2}{4}\right]dy$

Volume = of Solid. $\boxed{\int_0^6 \frac{3y^4}{16}\,dy}$

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CALCULUS AB

SECTION II, Part B

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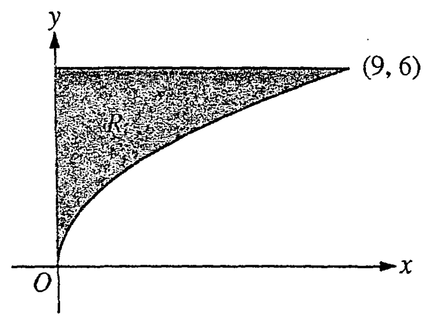

Work for problem 4(a)

$$R = \int_0^9 (6-2\sqrt{x})^2\,dx = \int_0^9 (36 - 24x^{1/2} + 4x)\,dx$$

$$= \left[36x - 16x^{3/2} + 2x^2\right]_0^9$$

$$= (324 - 432 + 162) - (0)$$

$$= (-108 + 162)$$

$$= \boxed{54 \text{ un.}^2}$$

$36 \times 9 = 324$

$81 \times 9 = 729$

$\sqrt{729} = \sqrt{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3} = 27$

27×16: 162, 270, 432

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Continue problem 4 on page 11.

4B2

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$\boxed{V = \pi\int_0^9 \left[(7-2\sqrt{x})^2 - (1)^2\right]dx}$$

Work for problem 4(c)

$(y)^2 = (2\sqrt{x})^2$

$y^2 = 4x$

$x = \frac{y^2}{4}$

Height of rectangle $= 3\left(\frac{y^2}{4}\right)$

Base $= \frac{y^2}{4}$

Area of rectangle $= 3\left(\frac{y^2}{4}\right)\left(\frac{y^2}{4}\right)$

$= \frac{3y^4}{16}$

$$\boxed{\text{Volume} = \int_0^6 \left(\frac{3y^4}{16}\right)dy}$$

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$4C_1$

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CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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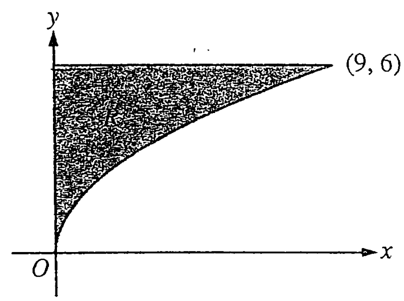

Work for problem 4(a)

$2(x)^{1/2 - 2}$

$\cancel{2} \cdot \frac{1}{\cancel{2}} x^{-\frac{1}{2}}$

$\int_0^9 6 - 2\sqrt{x}\, dx$

$0 - x^{-\frac{1}{2}} \Big|_0^9 = \frac{1}{\sqrt{x}} \Big|_0^9 = \frac{1}{\sqrt{9}} - \frac{1}{\sqrt{0}} = \boxed{\frac{1}{3}}$

$\boxed{R = \frac{1}{3}}$

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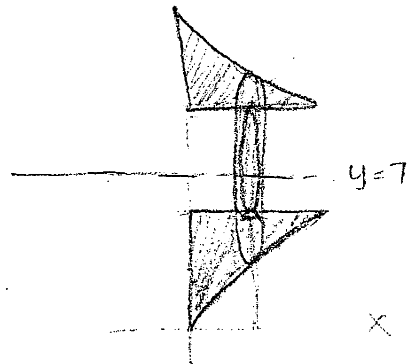
Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)



$\pi\int_0^9 (2\sqrt{x}-7)^2-(6-7)^2\,dx$

$\pi\int_0^9 (4x+49-28\sqrt{x})-(1)\,dx$

$\pi\int_0^9 4x+48-28\sqrt{x}\,dx$

$\pi\left(2x^2+48x-28\cdot\frac{2}{3}(x)^{3/2}\right)\Big|_0^9$ $(x)^{1/2+1}$ $\frac{3}{2}$

$\pi\left(2\cdot 81+48\cdot 9-\frac{56}{3}(9)^{3/2}\right)$

$\pi(90)=\boxed{90\pi}$

$\sqrt{9}\sqrt{9}\sqrt{9}$ $3\cdot3\cdot3$

$\sqrt{9^3}$ 9 27

162
422
584
−494
90

Work for problem 4(c)

$\pi\int_0^6 3\left((2\sqrt{x})^2-(6)^2\right)dx$

$\pi\int_0^6 3(4x-36)\,dx$

$\boxed{\pi\int_0^6 (12x-108)\,dx}$

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Question 4

Overview

In this problem students were given the graph of a region R bounded on the left by the y-axis, below by the curve $y = 2\sqrt{x}$, and above by the line $y = 6$. In part (a) students were asked to find the area of R, requiring an appropriate integral (or difference of integrals), antiderivative and evaluation. Part (b) asked for an integral expression that gives the volume of the solid obtained by revolving R about the line $y = 7$. This is found by integrating cross-sectional areas that correspond to washers with outer radius $7 - 2\sqrt{x}$ and inner radius 1, where $0 \le x \le 9$. Part (c) asked for an integral expression for the volume of a solid whose base is the region R and whose cross sections perpendicular to the y-axis are rectangles of height three times the lengths of their bases in R. Students needed to find the cross-sectional area function in terms of y and use this as the integrand in an integral with lower limit $y = 0$ and upper limit $y = 6$.

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: no points in part (a), 3 points in part (b), and 3 points in part (c). In part (a) the integrand is shown as the square of the expected integrand, so the student was not eligible for any points. In parts (b) and (c), the student's work is correct.

Sample: 4C
Score: 4

The student earned 4 points: 1 point in part (a), 3 points in part (b), and no points in part (c). In part (a) the student's integrand is correct, but the antiderivative is incorrect; the student differentiated rather than antidifferentiated. No additional points were earned in part (a). In part (b) the student presents an integral in the first line of the solution that earned all 3 points. The student works with the integral, making no errors in lines two and three, and finding an antiderivative in line four. The student's work in lines four and beyond was not evaluated since the question asked for an integral expression only, not for the value of the integral. In part (c) the student's integral was not eligible for any points.