AP® CALCULUS BC
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t)\,dt = 142.274$ or 142.275 cubic feet	$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$
(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.	1 : answer
(c) $h(0) = 0$ For $0 < t \le 6$, $h(t) = h(0) + \int_0^t g(s)\,ds = 0 + \int_0^t 0\,ds = 0.$ For $6 < t \le 7$, $h(t) = h(6) + \int_6^t g(s)\,ds = 0 + \int_6^t 125\,ds = 125(t-6).$ For $7 < t \le 9$, $h(t) = h(7) + \int_7^t g(s)\,ds = 125 + \int_7^t 108\,ds = 125 + 108(t-7).$ Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 125 + 108(t-7) & \text{for } 7 < t \le 9 \end{cases}$	$3 : \begin{cases} 1 : h(t) \text{ for } 0 \le t \le 6 \\ 1 : h(t) \text{ for } 6 < t \le 7 \\ 1 : h(t) \text{ for } 7 < t \le 9 \end{cases}$
(d) Amount of snow is $\int_0^9 f(t)\,dt - h(9) = 26.334$ or 26.335 cubic feet.	$3 : \begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

1A1

A graphing calculator is required for some problems or parts of problems.

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Work for problem 1(a)

$$\int_0^6 f(t)\,dt$$

$$\int_0^6 7te^{\cos t}\,dt$$

$$= 142.275 \text{ ft}^3$$

Work for problem 1(b)

$f(t) - g(t)$ at 8 am

$7te^{\cos t} - 108$ cubic feet per hour

$7(8)e^{\cos 8} - 108$

$= -59.583$ ft^3/hr

Continue problem 1 on page

Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \le t \le 6 \\ 125(t-6) & \text{for } 6 < t \le 7 \\ 108(t-7)+125 & \text{for } 7 < t \le 9 \end{cases}$$

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Work for problem 1(d)

$$\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$$

$$\int_0^9 7te^{\cos t}\,dt - h(t)\Big|_0^9$$

$$367.334 - (125 + 216)$$

$= 26.334$ ft^3 of snow are on the driveway at 9 am.

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CALCULUS BC

SECTION II, Part A

Time—45 minutes

Number of problems—3

1B, 1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Rate of accumalation of snow = $7te^{\cos t}$

Accumulation at 6 A.M = $\int_0^6 (7te^{\cos t})\,dt$

$\approx 142.275 \text{ ft}^3$

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Work for problem 1(b)

Volume of snow at 8 A.M = $7te^{\cos t} - 108$

$\frac{dV}{dt} = (7t)(e^{\cos t} \cdot -\sin t) + (7)(e^{\cos t})$

$\frac{dV}{dt} = 7t(-e^{\cos t}\sin t) + 7e^{\cos t}$

$\frac{dV}{dt} = -7te^{\cos t}\sin t + 7e^{\cos t}$

At $t = 8$, $\frac{dV}{dt} \approx -41.8496 \text{ ft}^3/\text{hr}$

Continue problem 1 on page 5

Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125t & \text{for } 6 \le t < 7 \\ 108t & \text{for } 7 \le t \le 9 \end{cases}$$

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Work for problem 1(d)

Total amount of snow falling from $0 \le t \le 9$

$= \int_0^9 (7te^{\cos t})\,dt \approx 367.33461 \text{ ft}^3$

From $6 \le t < 7$, Janet removed:

$\int_6^7 125\,dt = 125 \text{ ft}^3$

From $7 \le t \le 9$, Janet removed

$\int_7^9 108\,dt = 216 \text{ ft}^3$

So, at $t=9$, total snow $= (367.33461) - (125) - (216) \approx 26.335 \text{ ft}^3$

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CALCULUS AB

SECTION II, Part A

Time—45 minutes

Number of problems—3

1C1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$f(t) = 7te^{\cos t}$

$\int_0^6 (7te^{\cos t})\,dt = 142.275 \text{ ft}^3$

Work for problem 1(b)

$f(8) = 7(8)e^{\cos 8}$

$= 48.417 \text{ ft}^2/h$

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Continue problem 1 on page 5.

Work for problem 1(c)

$$\int_6^7 125\,dt = 125$$

$$\int_7^9 108\,dt = 216$$

$$h(t)\begin{cases} 0, & 0 \le t < 6 \\ 125, & 6 \le t < 7 \\ 216, & 7 \le t \le 9 \end{cases}$$

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Work for problem 1(d)

$$\int_0^9 (7te^{\cos t})\,dt = 367.335 \text{ ft}^2$$

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Question 1

Overview

This problem supplied two rate functions related to the amount of snow on Janet's driveway during a nine-hour period. One function f, given by $f(t) = 7te^{\cos t}$, measured in cubic feet per hour, models the rate of accumulation on the driveway for t between 0 and 9 hours after midnight. A second function, g, is a step function that gives the rate at which Janet removes snow from the driveway during this period. For part (a) students needed to use the definite integral $\int_0^6 f(t)\,dt$ to calculate the accumulation of snow on the driveway by 6 A.M. — integrating the rate of accumulation of snow over a time interval gives the net accumulation of snow during that time period. Part (b) asked for the rate of change of the volume of snow on the driveway at 8 A.M.; students needed to recognize this as the difference $f(8) - g(8)$ between the rate of accumulation and the rate of removal at time $t = 8$. Part (c) asked the students to recover a function h measuring the total amount of snow removed from the driveway for t between 0 and 9 hours after midnight. Students needed to integrate to obtain a piecewise-linear expression for h from the step function g. Part (d) asked for the amount of snow on the driveway at 9 A.M., which required students to compute the difference of two integrals, $\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student's work is correct. In part (b) the student works with f', rather than f and g. The student's numeric answer is incorrect. In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The second point was not earned since the student reports that the linear expression is $125t$. The student does not use the initial condition that $h(7) = 125$ and does not horizontally translate the linear expression, so the third point was not earned. In part (d) the student's work is correct.

Sample: 1C
Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student's work is correct. In part (b) the student does not subtract $g(8)$ from the evaluation of $f(8)$. In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The student presents constant functions for the other intervals and did not earn the other two points. In part (d) the student earned the point for the correct integral expression.