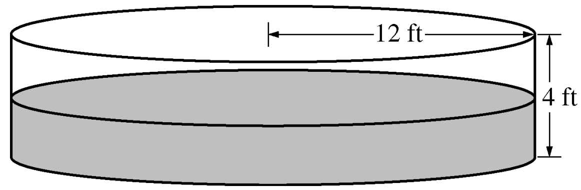
AP® CALCULUS BC
2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0.$ During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of $t.$ During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}.$ (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h.$)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

1 : answer

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\dfrac{dV}{dt} = 144\pi \dfrac{dh}{dt}$

$\left.\dfrac{dh}{dt}\right|_{t=8} = \dfrac{1}{144\pi} \cdot \left.\dfrac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft}/\text{hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{ equation relating } \dfrac{dV}{dt} \text{ and } \dfrac{dh}{dt} \\ 1 : \left.\dfrac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft}/\text{hr} \end{cases}$

3A1

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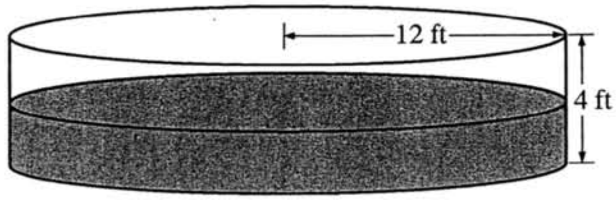

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Work for problem 3(a)

WATER ADDED INTO POOL $= \int_0^{12} P(t)\,dt \approx 4(46+57+62) = \boxed{660\ ft^3}$

ABOUT $\underline{660}\ ft^3$ OF WATER ARE ADDED TO THE POOL FROM $t = 0h$ TO $t = 12h$

Work for problem 3(b)

WATER LEAKED $= \int_0^{12} R(t)\,dt = \int_0^{12} (25e^{-.05t})\,dt = \boxed{225.594\ ft^3}$

$\underline{225.594}\ ft^3$ OF WATER LEAK FROM THE POOL FROM $t = 0h$ TO $t = 12h$

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Continue problem 3 on page 9.

Work for problem 3(c)

$V = \binom{\text{INITIAL}}{\text{WATER}} + (\text{WATER IN}) - (\text{WATER OUT}) = 1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1660 - 225.594 =$

$\boxed{= 1434.406\ ft^3}$

$\Downarrow$

$\boxed{1434\ ft^3}$

THE VOLUME OF WATER IN THE POOL AT TIME $t = 12\ h$ IS ABOUT $\underline{1434\ ft^3}$

Work for problem 3(d)

$V = \pi r^2 h$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$43.242 = 452.389 \frac{dh}{dt}$

$\frac{dh}{dt} = .096\ ft/hour$

$\frac{dV}{dt} = (\text{RATE WATER IN}) - (\text{RATE WATER OUT})$

$\frac{dV}{dt} = P(8) - R(8) = 43.242\ ft^3/hour$

AT $t = 8h$, THE VOLUME IN THE TANK IS INCREASING AT $\underline{43.242}\ ft^3/hour$

AT $t = 8h$, THE WATER LEVEL IS RISING AT $\underline{.096}\ ft/hour$

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END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

3B1

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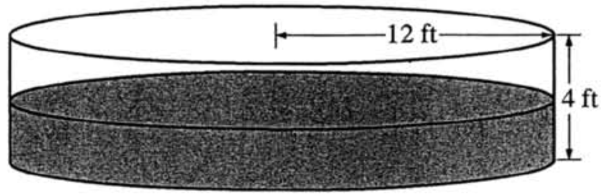

Work for problem 3(a)

Midpoints are $t = 2, 6, 10$, $p(t) = 46, 57, 62$.

$\text{Sum} = 4(46+57+62) = 660$.

Work for problem 3(b)

$V_{\text{water leaking out}} = \int_0^{12} R(t)\,dt = \int_0^{12} 25e^{-0.05t}\,dt$.

$= 225.594$

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Continue problem 3 on page 9.

3B2

Work for problem 3(c)

$V = V_{pumped} - V_{leak}$

$= 660 - 225.6 = 434.4$ cubic feet.

≈ 434 cubic feet.

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Work for problem 3(d)

$\frac{d}{dt}[p(t) - R(t)] = \frac{d}{dt}(80 - 25e^{-0.05t})$

$= -25 \cdot (-0.05t) \cdot e^{-0.05t - 1}$

since $t = 8 \rightarrow +25 \times 0.05 \times 8 \times e^{-0.05 \times 8 - 1}$.

$= 2.467$.

Thus, the volume of water is increasing at the rate of $2.467\ ft^3/h$ at $t = 8$

~~$V = \pi r^2 h = 2.467$~~. $V = \pi r^2 h$ $\quad \frac{dV}{dt} = \pi r^2 \cdot \frac{dh}{dt}$.

$\frac{dh}{dt} = \frac{2.467}{\pi \cdot 12^2} = 0.0055 ft$.

The water level is rising at the rate of $0.0055\ ft/h$ at $t = 8$.

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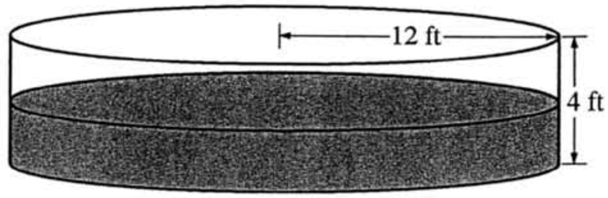

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Work for problem 3(a)

The approximate total amount of water

$$= \frac{(0+53)\times 4}{2} + \frac{(53+60)\times 4}{2} + \frac{(60+63)\times 4}{2}$$

$$= 289 \times 2$$

$$= 578 \text{ cubic feet}$$

Work for problem 3(b)

The total amount of water leaking out

$$= \int_0^{12} 25e^{-0.05t}\,dt = 255.594 \text{ cubic feet}$$

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Continue problem 3 on page 9.

3C2

Work for problem 3(c)

The approximate volume of water in time $t=12$

$= 1000 + 578 - 255.594$

$= 1322$ cubic feet

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Work for problem 3(d)

The rate $= \frac{d}{dt}(60 - 25e^{-0.05t}) = 1.25 \cdot e^{-0.05t}$

$= 0.8379$ cubic feet per second

The rate of rising $= \frac{d}{dt}(1.25 \cdot e^{-0.05t})$

$= -0.0625 \cdot e^{-0.05t}$

$= -0.419$ cubic feet per square second

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END OF PART A OF SECTION II

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Question 3

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), no points in part (c), and 2 points in part (d). In parts (a) and (b), the student's work is correct. In part (c) the student does not use the initial condition, and the point was not earned. In part (d) the student's presented value for $V'(8)$ is incorrect. The relationship between $\frac{dV}{dt}$ and $\frac{dh}{dt}$ is correct, and the value of $\frac{dh}{dt}$ is consistent with the student's $V'(8)$. The second and third points were earned. The units on $\frac{dh}{dt}$ are incorrect.

Sample: 3C
Score: 3

The student earned 3 points: no points in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student does not use a midpoint Riemann sum. In part (b) the student's work is correct. In part (c) the student correctly combines the results from parts (a) and (b) along with the initial condition. In part (d) the student's work is incorrect.