

AP[®] CALCULUS AB
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet’s driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

- (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
 (b) Find the rate of change of the volume of snow on the driveway at 8 A.M.
 (c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.
 (d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t) dt = 142.274$ or 142.275 cubic feet

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \leq 6$, $h(t) = h(0) + \int_0^t g(s) ds = 0 + \int_0^t 0 ds = 0$.

For $6 < t \leq 7$, $h(t) = h(6) + \int_6^t g(s) ds = 0 + \int_6^t 125 ds = 125(t - 6)$.

For $7 < t \leq 9$, $h(t) = h(7) + \int_7^t g(s) ds = 125 + \int_7^t 108 ds = 125 + 108(t - 7)$.

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 6 \\ 125(t - 6) & \text{for } 6 < t \leq 7 \\ 125 + 108(t - 7) & \text{for } 7 < t \leq 9 \end{cases}$

3 : $\begin{cases} 1 : h(t) \text{ for } 0 \leq t \leq 6 \\ 1 : h(t) \text{ for } 6 < t \leq 7 \\ 1 : h(t) \text{ for } 7 < t \leq 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t) dt - h(9) = 26.334$ or 26.335 cubic feet.

3 : $\begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$



CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

1A,

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$\int_0^6 f(t) dt$$
$$\int_0^6 7te^{\cos t} dt$$
$$= 142,275 \text{ ft}^3$$

Work for problem 1(b)

$$f(t) - g(t) \quad \text{at 8 am}$$
$$7te^{\cos t} - 108 \quad \text{cubic-feet per hour}$$
$$7(8)e^{\cos 8} - 108$$
$$= -59.583 \quad \text{ft}^3/\text{hr}$$

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Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 6 \\ 125(t-6) & \text{for } 6 < t \leq 7 \\ 108(t-7) + 125 & \text{for } 7 < t \leq 9 \end{cases}$$

Work for problem 1(d)

$$\begin{aligned} & \int_0^9 f(t) dt - \int_0^9 g(t) dt \\ & \int_0^9 7te^{\cos t} dt - h(t) \Big|_0^9 \\ & 367.334 - (125 + 216) \\ & = 26.334 \text{ ft}^3 \text{ of snow are on the driveway} \\ & \text{at 9 am.} \end{aligned}$$

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CALCULUS BC
SECTION II, Part A

Time—45 minutes

Number of problems—3

B,

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Rate of accumulation of snow = $7te^{\cos t}$

$$\text{Accumulation at 6 A.M.} = \int_0^6 (7te^{\cos t}) dt$$

$$\approx 142.275 \text{ ft}^3$$

Work for problem 1(b)

Volume of snow at 8 A.M. = $7te^{\cos t} - 108$

$$\frac{dV}{dt} = (7t)(e^{\cos t} \cdot -\sin t) + (7)(e^{\cos t})$$

$$\frac{dV}{dt} = 7t(-e^{\cos t} \sin t) + 7e^{\cos t}$$

$$\frac{dV}{dt} = -7te^{\cos t} \sin t + 7e^{\cos t}$$

$$\text{At } t=8, \frac{dV}{dt} \approx -41.8496 \text{ ft}^3/\text{hr}$$

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Work for problem 1(c)

$$h(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125t & \text{for } 6 \leq t < 7 \\ 108t & \text{for } 7 \leq t \leq 9 \end{cases}$$

Work for problem 1(d)

Total amount of snow falling from $0 \leq t \leq 9$

$$= \int_0^9 (7te^{\cos t}) dt \approx 367.33461 \text{ ft}^3$$

From $6 \leq t < 7$, Janet removed:

$$\int_6^7 125 dt = 125 \text{ ft}^3$$

From $7 \leq t \leq 9$, Janet removed:

$$\int_7^9 108 dt = 216 \text{ ft}^3$$

So, at $t=9$, total snow = $(367.33461) - (125) - (216) \approx 26.335 \text{ ft}^3$

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CALCULUS AB
SECTION II, Part A

Time—45 minutes

Number of problems—3

1C1

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

$$f(t) = 7te^{\cos t}$$

$$\int_0^6 (7te^{\cos t}) dt = 742.275 \text{ ft}^3$$

Work for problem 1(b)

$$f(s) = 7(s)e^{\cos s}$$

$$= 48.417 \text{ ft}^2/\text{h}$$

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Work for problem 1(c)

$$\int_6^7 125 dt = 125$$

$$\int_7^9 108 dt = 216$$

$$h(t) \begin{cases} 0, & 0 \leq t < 6 \\ 125, & 6 \leq t < 7 \\ 216, & 7 \leq t \leq 9 \end{cases}$$

Work for problem 1(d)

$$\int_0^9 (7te^{\cos t}) dt = 367.335 ft^2$$

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AP[®] CALCULUS AB
2010 SCORING COMMENTARY

Question 1

Overview

This problem supplied two rate functions related to the amount of snow on Janet’s driveway during a nine-hour period. One function f , given by $f(t) = 7te^{\cos t}$, measured in cubic feet per hour, models the rate of accumulation on the driveway for t between 0 and 9 hours after midnight. A second function, g , is a step function that gives the rate at which Janet removes snow from the driveway during this period. For part (a) students needed to use the definite integral $\int_0^6 f(t) dt$ to calculate the accumulation of snow on the driveway by 6 A.M. — integrating the rate of accumulation of snow over a time interval gives the net accumulation of snow during that time period. Part (b) asked for the rate of change of the volume of snow on the driveway at 8 A.M.; students needed to recognize this as the difference $f(8) - g(8)$ between the rate of accumulation and the rate of removal at time $t = 8$. Part (c) asked the students to recover a function h measuring the total amount of snow removed from the driveway for t between 0 and 9 hours after midnight. Students needed to integrate to obtain a piecewise-linear expression for h from the step function g . Part (d) asked for the amount of snow on the driveway at 9 A.M., which required students to compute the difference of two integrals, $\int_0^9 f(t) dt - \int_0^9 g(t) dt$.

Sample: 1A

Score: 9

The student earned all 9 points.

Sample: 1B

Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 3 points in part (d). In part (a) the student’s work is correct. In part (b) the student works with f' , rather than f and g . The student’s numeric answer is incorrect. In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The second point was not earned since the student reports that the linear expression is $125t$. The student does not use the initial condition that $h(7) = 125$ and does not horizontally translate the linear expression, so the third point was not earned. In part (d) the student’s work is correct.

Sample: 1C

Score: 4

The student earned 4 points: 2 points in part (a), no points in part (b), 1 point in part (c), and 1 point in part (d). In part (a) the student’s work is correct. In part (b) the student does not subtract $g(8)$ from the evaluation of $f(8)$. In part (c) the student earned the first point for correctly identifying $h(t) = 0$ on the interval from 0 to 6. The student presents constant functions for the other intervals and did not earn the other two points. In part (d) the student earned the point for the correct integral expression.