CollegeBoard AP

AP® Calculus AB
2010 Scoring Guidelines
Form B

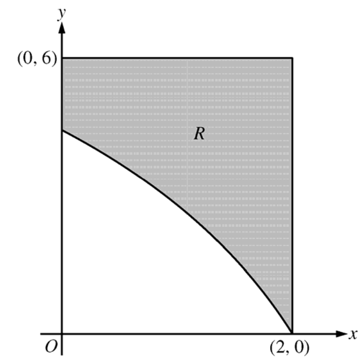
Question 1

In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.



1 : Correct limits in an integral in (a), (b), or (c)

(a) $\int_0^2 (6 - 4\ln(3-x))\,dx = 6.816$ or 6.817

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi\int_0^2 \left((8 - 4\ln(3-x))^2 - (8-6)^2\right) dx$
$= 168.179$ or 168.180

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^2 (6 - 4\ln(3-x))^2\,dx = 26.266$ or 26.267

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 2

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

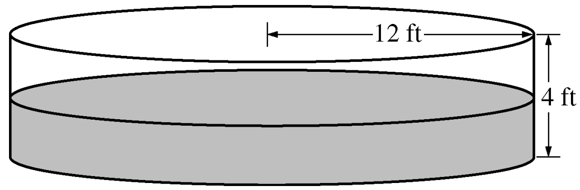
(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

(a) The graph of g has a horizontal tangent line when $g'(x) = 0$. This occurs at $x = 0.163$ and $x = 0.359$.	$2: \begin{cases} 1: \text{sets } g'(x) = 0 \\ 1: \text{answer} \end{cases}$
(b) $g''(x) = 0$ at $x = 0.129458$ and $x = 0.222734$ The graph of g is concave down on $(0.1295, 0.2227)$ because $g''(x) < 0$ on this interval.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) $g'(0.3) = -0.472161$ $g(0.3) = 2 + \int_1^{0.3} g'(x)\,dx = 1.546007$ An equation for the line tangent to the graph of g is $y = 1.546 - 0.472(x - 0.3)$.	$4: \begin{cases} 1: g'(0.3) \\ 1: \text{integral expression} \\ 1: g(0.3) \\ 1: \text{equation} \end{cases}$
(d) $g''(x) > 0$ for $0.3 < x < 1$ Therefore the line tangent to the graph of g at $x = 0.3$ lies below the graph of g for $0.3 < x < 1$.	1 : answer with reason

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0.$ During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of $t.$ During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}.$ (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h.$)

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

(c) Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t)\,dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t)\,dt = 225.594 \text{ ft}^3$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t)\,dt - \int_0^{12} R(t)\,dt = 1434.406$

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3.

$1 : \text{answer}$

(d) $V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241 \text{ or } 43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

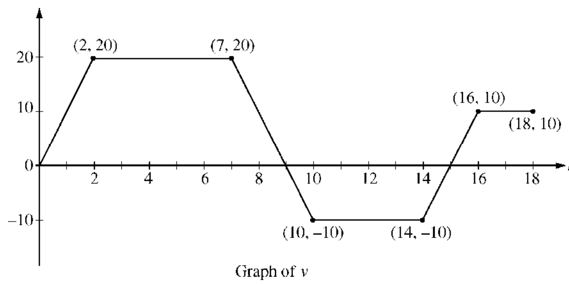
$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left.\frac{dh}{dt}\right|_{t=8} = \frac{1}{144\pi} \cdot \left.\frac{dV}{dt}\right|_{t=8} = 0.095 \text{ or } 0.096 \text{ ft/hr}$

$4 : \begin{cases} 1 : V'(8) \\ 1 : \text{ equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left.\frac{dh}{dt}\right|_{t=8} \\ 1 : \text{units of ft}^3/\text{hr and ft/hr} \end{cases}$

Question 4

A squirrel starts at building A at time $t = 0$ and travels along a straight wire connected to building B. For $0 \le t \le 18$, the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.



(a) At what times in the interval $0 < t < 18$, if any, does the squirrel change direction? Give a reason for your answer.

(b) At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building A? How far from building A is the squirrel at this time?

(c) Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

(d) Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building A that are valid for the time interval $7 < t < 10$.

(a) The squirrel changes direction whenever its velocity changes sign. This occurs at $t = 9$ and $t = 15$.

$2 : \begin{cases} 1 : t\text{-values} \\ 1 : \text{explanation} \end{cases}$

(b) Velocity is 0 at $t = 0$, $t = 9$, and $t = 15$.

t	position at time t
0	0
9	$\frac{9+5}{2} \cdot 20 = 140$
15	$140 - \frac{6+4}{2} \cdot 10 = 90$
18	$90 + \frac{3+2}{2} \cdot 10 = 115$

The squirrel is farthest from building A at time $t = 9$; its greatest distance from the building is 140.

$2 : \begin{cases} 1 : \text{identifies candidates} \\ 1 : \text{answers} \end{cases}$

(c) The total distance traveled is $\int_0^{18} |v(t)|\, dt = 140 + 50 + 25 = 215$.

1 : answer

(d) For $7 < t < 10$, $a(t) = \dfrac{20 - (-10)}{7 - 10} = -10$

$$v(t) = 20 - 10(t - 7) = -10t + 90$$

$$x(7) = \frac{7+5}{2} \cdot 20 = 120$$

$$x(t) = x(7) + \int_7^t (-10u + 90)\, du$$

$$= 120 + \left(-5u^2 + 90u\right)\Big|_{u=7}^{u=t}$$

$$= -5t^2 + 90t - 265$$

$4 : \begin{cases} 1 : a(t) \\ 1 : v(t) \\ 2 : x(t) \end{cases}$

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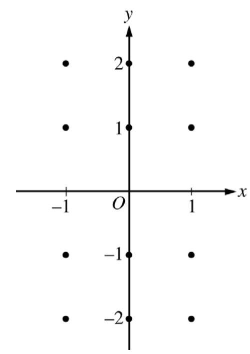
2010 SCORING GUIDELINES (Form B)

Question 5

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

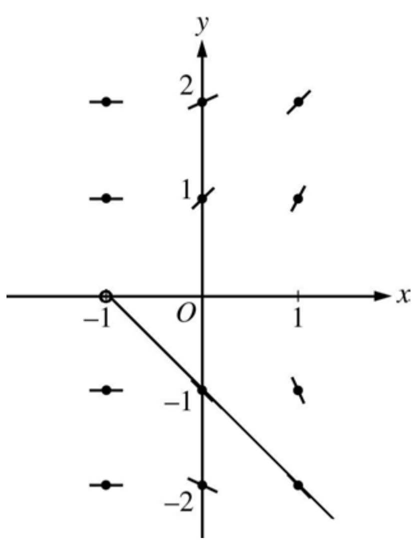
(**Note: Use the axes provided in the exam booklet.**)



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the *xy*-plane for which $y \neq 0$. Describe all points in the *xy*-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

(a)



$3: \begin{cases} 1: \text{zero slopes} \\ 1: \text{nonzero slopes} \\ 1: \text{solution curve through } (0, -1) \end{cases}$

(b) $-1 = \frac{x+1}{y} \Rightarrow y = -x - 1$

$\frac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

1 : description

(c) $\int y\, dy = \int (x+1)\, dx$

$\frac{y^2}{2} = \frac{x^2}{2} + x + C$

$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

Since the solution goes through $(0, -2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

$5: \begin{cases} 1: \text{separates variables} \\ 1: \text{antiderivatives} \\ 1: \text{constant of integration} \\ 1: \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

Question 6

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3$.

(a) For $0 \le t \le 6$, find all times t during which particle R is moving to the right.

(b) For $0 \le t \le 6$, find all times t during which the two particles travel in opposite directions.

(c) Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

(d) Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3$.

(a) $r'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$

$r'(t) = 0$ when $t = 1$ and $t = 3$

$r'(t) > 0$ for $0 < t < 1$ and $3 < t < 6$

$r'(t) < 0$ for $1 < t < 3$

Therefore R is moving to the right for $0 < t < 1$ and $3 < t < 6$.

$2 : \begin{cases} 1 : r'(t) \\ 1 : \text{answer} \end{cases}$

(b) $p'(t) = -2 \cdot \frac{\pi}{4}\sin\left(\frac{\pi}{4}t\right)$

$p'(t) = 0$ when $t = 0$ and $t = 4$

$p'(t) < 0$ for $0 < t < 4$

$p'(t) > 0$ for $4 < t < 6$

Therefore the particles travel in opposite directions for $0 < t < 1$ and $3 < t < 4$.

$3 : \begin{cases} 1 : p'(t) \\ 1 : \text{sign analysis for } p'(t) \\ 1 : \text{answer} \end{cases}$

(c) $p''(t) = -2 \cdot \frac{\pi}{4} \cdot \frac{\pi}{4}\cos\left(\frac{\pi}{4}t\right)$

$p''(3) = -2\left(\frac{\pi}{4}\right)^2 \cos\left(\frac{3\pi}{4}\right) = \frac{\pi^2}{8} \cdot \frac{\sqrt{2}}{2} > 0$

$p'(3) < 0$

Therefore particle P is slowing down at time $t = 3$.

$2 : \begin{cases} 1 : p''(3) \\ 1 : \text{answer with reason} \end{cases}$

(d) $\frac{1}{2}\int_1^3 \left| p(t) - r(t) \right| dt$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$