Question 5

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x+1}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1)$.

(**Note: Use the axes provided in the exam booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\dfrac{dy}{dx} = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

(a)
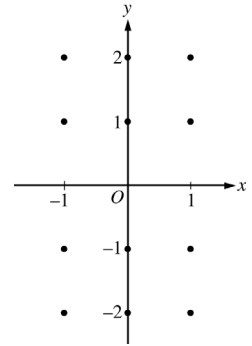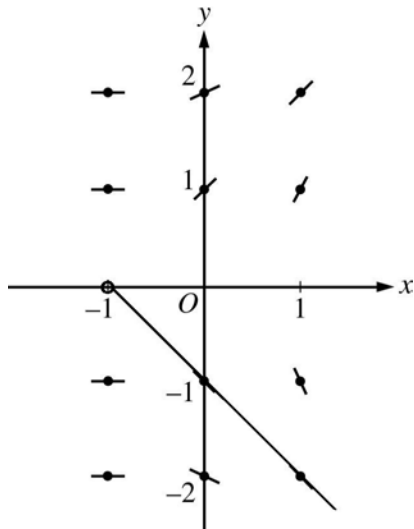
$$
\left.3 : \begin{cases} 1 : \text{zero slopes} \\ 1 : \text{nonzero slopes} \\ 1 : \text{solution curve through } (0, -1) \end{cases}\right.
$$

(b) $-1 = \dfrac{x+1}{y} \Rightarrow y = -x - 1$

$\dfrac{dy}{dx} = -1$ for all (x, y) with $y = -x - 1$ and $y \neq 0$

1 : description

(c) $\displaystyle \int y \, dy = \int (x+1) \, dx$

$\dfrac{y^2}{2} = \dfrac{x^2}{2} + x + C$

$\dfrac{(-2)^2}{2} = \dfrac{0^2}{2} + 0 + C \Rightarrow C = 2$

$y^2 = x^2 + 2x + 4$

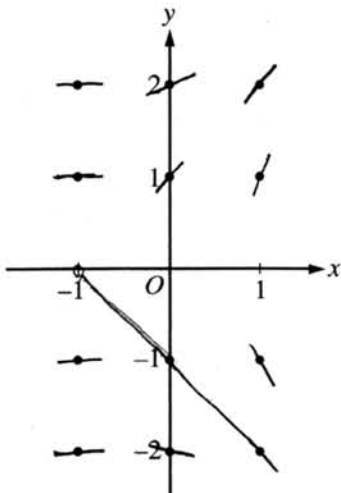Since the solution goes through $(0, -2)$, y must be negative. Therefore $y = -\sqrt{x^2 + 2x + 4}$.

$$
\left.5 : \begin{cases} 1 : \text{separates variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}\right.
$$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

Work for problem 5(a)

Work for problem 5(b)

$$\frac{dy}{dx} = \frac{x+1}{y} = -1 \quad \Rightarrow \quad x+1 = -y \quad \Rightarrow \quad x+y = -1$$

except point$(-1, 0)$, points $(x, -1-x)$

are all solution to $\frac{dv}{dx} = -1$.

They are on the line $y = -1-x$

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Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$dy \cdot y = dx \cdot x+1$$

$$\int y \cdot dy = \int x+1 \, dx$$

$$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C \qquad \text{passes through } f(0) = -2$$

$$\frac{1}{2} \times 4 = C \Rightarrow C = 2$$

$$y = -\sqrt{x^2 + 2x + 4}$$

Work for problem 5(a)

Work for problem 5(b)

$$\frac{dy}{dx} = -1 \quad \text{when}$$

$$0 \leq x \leq 1 \quad \text{and} \quad -2 \leq y \leq -1$$

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Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y} \qquad f(0) = -2$$

$$y\,dy = x+1\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + C$$

$$\frac{(-2)^2}{2} = \frac{0^2}{2} + 0 + C$$

$$\frac{4}{2} = C$$

$$2 = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} + x + 2$$

$$y^2 = 2\left(\frac{x^2}{2} + x + 2\right)$$

$$y^2 = x^2 + 2x + 4$$

$$y = \sqrt{x^2 + 2x + 4}$$

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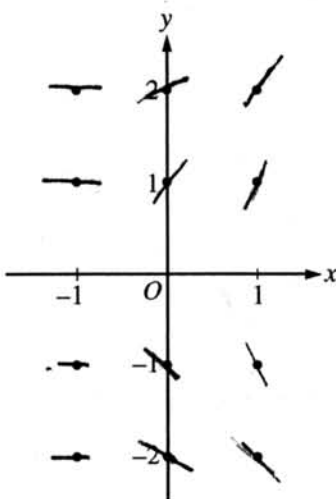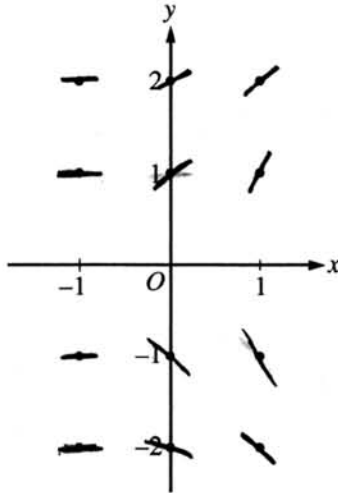Do not write beyond this border.

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Work for problem 5(a)

Work for problem 5(b)

$y = -x$

$\dfrac{dy}{dx} = -1$ means that the slope is equal to -1.

For all x, if $f(x) = -x$ $\dfrac{dy}{dx} = -1$

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NO CALCULATOR ALLOWED

Work for problem 5(c)

$$\frac{dy}{dx} = \frac{x+1}{y}$$

$$y \cdot dy = dx\,(x+1)$$

$$\int y \cdot dy = \int dx\,(x+1)$$

$$y = x + 1$$

$$f(0) = -2$$

$$dx = -2\,dy$$

$$\int dx = \int -2\,dy$$

$$x = -2y$$

$$y = f\left(-\frac{2}{x}\right)$$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), and 4 points in part (c). In part (a) the student's slope field is correct, but no solution curve is given. In part (b) the student's description is incorrect. In part (c) the student earned the first 4 points. Although the student uses the initial condition, the incorrect branch is chosen for the solution.

Sample: 5C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), and 1 point in part (c). In part (a) the student's slope field is correct, but no solution curve is given. In part (b) the student's description is incorrect. In part (c) the student earned the point for separation of variables. The antiderivatives are not correct, so the student was not eligible for additional points.