Question 4

Intent of Question

The primary goals of this question were to evaluate a student's ability to (1) identify and compute an appropriate confidence interval after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to make an inference about whether or not a council member's belief is supported.

Solution

Part (a):

Step 1: Identify the appropriate confidence interval by name or formula and check for appropriate conditions.

The two-sample *t* interval for $\mu_N - \mu_S$, the difference in population mean response times, is

$$\left(\bar{x}_N - \bar{x}_S\right) \pm t^* \sqrt{\frac{s_N^2}{n_N} + \frac{s_S^2}{n_S}}$$

where μ_N denotes the mean response for calls from the northern fire station and μ_S denotes the mean response for calls from the southern fire station.

Conditions: 1. Independent random samples
2. Large samples or normal population distributions

A random sample of 50 calls was selected from the northern fire station, independent of the random sample of 50 calls selected from the southern fire station.

The use of the two-sample *t* interval is reasonable because both sample sizes are large ($n_N = 50 > 30$ and $n_S = 50 > 30$), and by the central limit theorem, the sampling distributions for the two sample means are approximately normal. Therefore the sampling distribution of the difference of the sample means $\bar{x}_N - \bar{x}_S$ is approximately normal.

Step 2: Correct Mechanics

Unequal variances: Degrees of freedom = 96.

$$(4.3 - 5.3) \pm 1.985\sqrt{\frac{3.7^2}{50} + \frac{3.2^2}{50}}$$

$$-1.0 \pm 1.985 \times .6918$$

$$(-2.37, 0.37)$$

Step 3: Interpretation

Based on these samples, one can be 95 percent confident that the difference in the population mean response times (northern - southern) is between -2.37 minutes and 0.37 minutes.

Question 4 (continued)

Part (b):

Zero is within the 95 percent confidence interval of plausible values for the difference in population means. Therefore this confidence interval does not support the council member's belief that there is a difference in mean response times for the two fire stations.

Scoring

This problem is scored in four sections. Section 1 consists of part (a), step 1; section 2 consists of part (a), step 2; section 3 consists of part (a), step 3; and section 4 consists of part (b). Sections 1, 2, and 3 are scored as essentially correct (E) or incorrect (I), and section 4 is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1: Identify the appropriate confidence interval by name or formula and check for appropriate conditions.

Section 1 is scored as follows:

Essentially correct (E) if the student does both of the following:

- Indicates that the appropriate procedure is a *t* confidence interval for difference in means, either by name or formula.
- Verifies that the sample sizes are large enough to use this procedure by referencing a number (such as 25 or 30) *OR* by stating that 50 is large enough that the central limit theorem applies.

Incorrect (I) if the student omits at least one of the two elements above *OR* names or gives a formula for a two-sample *z* confidence interval instead of a two-sample *t* confidence interval.

Section 2: Correct Mechanics

Section 2 is scored as follows:

Essentially correct (E) if the student shows a correct confidence interval, either by displaying numbers in the formula or by writing the numerical interval.

Note: The following are acceptable solutions for section 2:

- The following degrees of freedom, t^* values, and confidence intervals are all acceptable.

Solutions to Step 2			
Procedure	**Degrees of freedom**	t^*	**Confidence interval for $\mu_N - \mu_S$**
Unequal variances	96	1.985	(-2.37, 0.37)
Conservative df	40 (using table)	2.021	(-2.40, 0.40)
Conservative df	49	2.010	(-2.39, 0.39)
Conservative df	50 (using table)	2.009	(-2.39, 0.39)
Pooled variance	98	1.984	(-2.37, 0.37)

Question 4 (continued)

- Students do not need to state explicitly the degrees of freedom used.
- It is acceptable if a student reports a confidence interval for $\mu_S - \mu_N$, in which case the signs are reversed.
- An identifiable minor arithmetic error will not necessarily change a score on section 2 from essentially correct to incorrect.

Incorrect (I) if the numerical interval given is incorrect because of a substantive statistical error, such as failing to square the standard deviations *OR* if the interval is missing or completely wrong.

Section 3: Interpretation

Section 3 is scored as follows:

Essentially correct (E) if the student correctly interprets the confidence interval (not the confidence level) in context. The interpretation must indicate that the interval is for a difference in mean response times. If the student gives an interpretation of *both* the confidence level and the confidence interval, both must be correct to be scored as essentially correct (E).

Incorrect (I) if a student gives an interpretation of the confidence level *instead of* the confidence interval *OR* if the interpretation of the confidence interval is wrong.

Note: The correct interpretation of the confidence interval for step 3 of part (a) may be found in part (b).

Section 4

Section 4 is scored as follows:

Essentially correct (E) if the student makes a correct conclusion in context, supported by the fact that zero is contained within the 95 percent confidence interval.

Partially correct (P) if the student makes a correct conclusion supported by the fact that zero is contained in the confidence interval, but one or both of the following occurs:
- The student omits the context.
- The student makes a statistically incorrect statement in the explanation, such as stating that the council member's belief is wrong. Such a statement is equivalent to accepting the null hypothesis of no difference in population mean response times.

Incorrect (I) if the student gives the incorrect conclusion *OR* makes no reference to a confidence interval.

Notes:
- The answer in part (b) needs to be consistent with the confidence interval given in part (a). If the interval does not cover zero, then part (b) must state that the interval does support the council member's belief because the interval does not contain zero.
- The response for part (b) may be found in the space allocated to part (a).

Question 4 (continued)

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (b) counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 1½ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.

4. One of the two fire stations in a certain town responds to calls in the northern half of the town, and the other fire station responds to calls in the southern half of the town. One of the town council members believes that the two fire stations have different mean response times. Response time is measured by the difference between the time an emergency call comes into the fire station and the time the first fire truck arrives at the scene of the fire.

Data were collected to investigate whether the council member's belief is correct. A random sample of 50 calls selected from the northern fire station had a mean response time of 4.3 minutes with a standard deviation of 3.7 minutes. A random sample of 50 calls selected from the southern fire station had a mean response time of 5.3 minutes with a standard deviation of 3.2 minutes.

(a) Construct and interpret a 95 percent confidence interval for the difference in mean response times between the two fire stations.

① We are trying to determine $\mu_1 - \mu_2$, the true difference in mean response times between the northern fire station (μ_1) and the southern fire station (μ_2). Our best guess is the difference in sample mean responses, $\bar{x}_1 - \bar{x}_2 = 4.3 - 5.3 = -1.0$. However, because of sampling variability, this is unlikely to be true. Instead, we will construct a 95% t-confidence interval for $\mu_1 - \mu_2$.

② Conditions: a) Independent random samples of calls to fire stations? Given.

b) Large samples of calls to fire stations? $n_1 = 50 > 30$ ✓

$n_2 = 50 > 30$ ✓

c) Samples < 10% Populations? Yes, assuming there are more than 500 calls made to northern fire station and 500 to southern fire station.

using df=40 in table

③ 95% t-CI $= -1.0 \pm 2.021\sqrt{\frac{3.7^2}{50} + \frac{3.2^2}{50}} = -1.0 \pm 1.3981 = (-2.3981, 0.3981)$

df = (smaller of n_1 or n_2) $- 1 = 50 - 1 = 49$

④ Thus, we are 95% confident that the interval -2.3981 minutes to 0.3981 minutes include's the true difference in mean response times between the northern fire station and the southern fire station.

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If you need more room for your work for part (a), use the space below.

4A2

(b) Does the confidence interval in part (a) support the council member's belief that the two fire stations have different mean response times? Explain.

The confidence interval in part (a) does not support the council member's belief that the two fire stations have different mean responce times because it includes the value 0. If the interval includes the value 0, it is possible that there is no true difference between the mean response times between the northern fire station and the southern fire station ($\mu_1 - \mu_2 = 0$).

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4B1

4. One of the two fire stations in a certain town responds to calls in the northern half of the town, and the other fire station responds to calls in the southern half of the town. One of the town council members believes that the two fire stations have different mean response times. Response time is measured by the difference between the time an emergency call comes into the fire station and the time the first fire truck arrives at the scene of the fire.

Data were collected to investigate whether the council member's belief is correct. A random sample of 50 calls selected from the northern fire station had a mean response time of 4.3 minutes with a standard deviation of 3.7 minutes. A random sample of 50 calls selected from the southern fire station had a mean response time of 5.3 minutes with a standard deviation of 3.2 minutes.

(a) Construct and interpret a 95 percent confidence interval for the difference in mean response times between the two fire stations.

These samples were both done randomly and according to the Central Limit Theorm these distributions will be approximentdly normal. We can safely proceed.

2 sample T Interval (-2.373, .3732)

We are 95 percent confident that we have captured the true mean for the difference in mean response times between the two fire stations. ~~From this interval~~

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4B2

If you need more room for your work for part (a), use the space below.

(b) Does the confidence interval in part (a) support the council member's belief that the two fire stations have different mean response times? Explain.

From the confidence interval we can not safely conclude that the council member was correct. Since both positive and negative values are included in this interval we can not tell whether one fire stations response time is faster, slower or the same.

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4. One of the two fire stations in a certain town responds to calls in the northern half of the town, and the other fire station responds to calls in the southern half of the town. One of the town council members believes that the two fire stations have different mean response times. Response time is measured by the difference between the time an emergency call comes into the fire station and the time the first fire truck arrives at the scene of the fire.

Data were collected to investigate whether the council member's belief is correct. A random sample of 50 calls selected from the northern fire station had a mean response time of 4.3 minutes with a standard deviation of 3.7 minutes. A random sample of 50 calls selected from the southern fire station had a mean response time of 5.3 minutes with a standard deviation of 3.2 minutes.

(a) Construct and interpret a 95 percent confidence interval for the difference in mean response times between the two fire stations.

1. 95% confidence interval for μ_1 = true mean of calls selected from the northern fire station

μ_2 = true mean of calls selected from the southern fire station

2. both are simple random samples and large sample size because $50 \geq 30$

3. (-2.373, .37323)

4. we are 95% confident that the difference in mean response times between the north and south fire station is between -2.373 and .37323

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4C2

If you need more room for your work for part (a), use the space below.

(b) Does the confidence interval in part (a) support the council member's belief that the two fire stations have different mean response times? Explain.

yes it does support the council member's belief that the two fire stations have different mean response times because there can be a possible difference of -2.373 to .37323 of the response times.

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Question 4

Overview

The primary goals of this question were to evaluate a student's ability to (1) identify and compute an appropriate confidence interval after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to make an inference about whether or not a council member's belief is supported.

Sample: 4A
Score: 4

The student does a very nice job of presenting every part of the problem. In part (a) the student explains the problem, defines notation used, names the *t* confidence interval for $\mu_1 - \mu_2$, and checks all the conditions. The mechanics are presented in detail, including a confidence interval formula with numbers, as well as the final confidence interval, either one of which would have been sufficient. Conservative degrees of freedom are used, and the student explains how to find the multiplier using the *t*-table in that situation. The student then gives a nice, clear interpretation of the confidence interval. Part (a), steps 1, 2, and 3 (scored as sections 1, 2, and 3) were scored as essentially correct. In part (b) the student makes the correct conclusion in context, justifies it by noting that the confidence interval includes the value 0, and explains why the value of 0 would imply that the council member's belief is not supported. Part (b) (scored as section 4) was scored as essentially correct. This exemplary answer, based on all four sections, was judged a complete response and earned a score of 4 points.

Sample: 4B
Score: 3

This response presents minimal information, but most of what is presented is relevant and correct. In part (a), step 1 (section 1) the student names the correct procedure but fails to explicitly check the sample size condition. The one incorrect statement the student makes is: "These samples were both done randomly and according to the Central Limit Theorem these distributions will be approximentally [*sic*] normal." The student does not explain that the Central Limit Theorem applies because the samples are large, nor does the student explain that the distribution that needs to be approximately normal is the sampling distribution of the difference in sample means. Therefore part (a), step 1 (section 1) was scored as incorrect. The student presents a correct confidence interval, so part (a), step 2 (section 2) was scored as essentially correct. The interpretation of the confidence interval includes one extra use of the word "mean" but is still a correct statement, and it is given in context. Although not ideal, it is acceptable that the numerical interval is omitted from the interpretation, so part (a), step 3 (section 3) was scored as essentially correct. In part (b) the student does not explicitly mention that 0 is contained in the confidence interval, but stating that "both positive and negative values are included in this interval" is equivalent. The correct conclusion and justification are provided in context, so part (b) (section 4) was scored as essentially correct. With one section incorrect and three sections essentially correct, the entire answer was judged a substantial response and earned a score of 3 points.

Sample: 4C
Score: 2

The information provided in part (a) is all correct, but some necessary information is missing. The student defines notation but does not name the procedure to be used or provide a formula for it, so part (a), step 1 (section 1) was scored as incorrect. However, all the other information needed for part (a) is provided. The

Question 4 (continued)

student checks that the sample size condition is met, provides a correct numerical confidence interval, and gives an appropriate interpretation of the confidence interval, in context, so part (a), steps 2 and 3 (sections 2 and 3) were scored as essentially correct. In part (b) the student gives the wrong conclusion, stating that the council member's belief is supported because some of the values in the interval support it, so part (b) (section 4) was scored as incorrect. With two sections incorrect and two sections essentially correct, the entire answer was judged a developing response and earned a score of 2 points.