Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) state and check appropriate conditions for inference in a study, (2) state the appropriate hypotheses for a statistical test, (3) identify and compute the appropriate test statistic, and (4) make a conclusion in the context of the problem.

Solution

Let A represent the cardiopump treatment, and let B represent the CPR treatment.

Let p_A = proportion of patients who will survive at least one year if treated with the cardiopump.
Let p_B = proportion of patients who will survive at least one year if treated with CPR.

Part (a):

Step 1: State the conditions for inference.

The conditions required for a two-sample *z* test of equal proportions for an experiment are:

1. Random assignment of treatments to subjects
2. Sufficiently large sample sizes

Step 2: Check the conditions.

1. If we assume that the relevant characteristics of people who have heart attacks on even-numbered and odd-numbered days are comparable, randomly assigning one treatment to be given on even-numbered days and the other to be given on odd-numbered days is a reasonable approximation to randomly assigning the two treatments to the available subjects.
2. The large sample condition is met because all of the following are at least 5 (or 10):

$$n_A\hat{p}_A = 37 \ge 5 \text{ or } 10,\ n_A(1-\hat{p}_A) = 717 \ge 5 \text{ or } 10$$
$$n_B\hat{p}_B = 15 \ge 5 \text{ or } 10,\ n_B(1-\hat{p}_B) = 731 \ge 5 \text{ or } 10$$

Part (b):

Step 1: State a correct pair of hypotheses.

$$H_0: p_A - p_B = 0 \text{ (or } p_A = p_B)$$
$$H_a: p_A - p_B > 0 \text{ (or } p_A > p_B)$$

Step 2: Identify a correct test by name or by formula.

Two-sample *z* test for proportions

OR

Question 3 (continued)

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_A} + \frac{1}{n_B}\right)}} = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_A} + \frac{\hat{p}(1-\hat{p})}{n_B}}} \text{ where } \hat{p} = \frac{n_A\hat{p}_A + n_B\hat{p}_B}{n_A + n_B}.$$

<u>Step 3</u>: Correct mechanics, including the value of the test statistic and p-value (or rejection region).

$$\hat{p}_A = \frac{37}{754} \approx 0.049 \qquad \hat{p}_B = \frac{15}{746} \approx 0.020 \qquad \hat{p} = \frac{37+15}{754+746} = \frac{52}{1500} \approx 0.035$$

$$z = \frac{\frac{37}{754} - \frac{15}{746}}{\sqrt{\frac{52}{1500}\left(1 - \frac{52}{1500}\right)\left(\frac{1}{754} + \frac{1}{746}\right)}} \approx 3.066$$

The p-value is 0.0011.

<u>Step 4</u>: State a correct conclusion in the context of the problem, using the result of the statistical test.

Because the p-value of 0.0011 is very small, that is, less than any reasonable significance level such as $\alpha = 0.01$, or $\alpha = 0.05$, we reject the null hypothesis. We have strong evidence to support the conclusion that the proportion of patients who survive when treated with the cardiopump is higher than the proportion of patients who survive when treated with CPR; that is, the survival rate is higher for patients treated with the cardiopump. (*OR*, If all of these patients had been assigned the cardiopump, we have strong evidence that the survival rate would be higher than if all of these patients had been assigned CPR.)

<u>Scoring</u>

This problem is scored in four sections. Section 1 consists of part (a). Section 2 consists of part (b), step 1. Section 3 consists of part (b), steps 2 and 3. Section 4 consists of part (b), step 4. Section 1 is scored as essentially correct (E), partially correct (P), or incorrect (I). Sections 2, 3, and 4 are each scored as essentially correct (E) or incorrect (I).

Section 1 [part (a)] is scored as follows:

Essentially correct (E) if the student correctly states and addresses randomization with reasonable justification *AND* correctly checks that the numbers of successes and failures are at least 5 or 10.

Partially correct (P) if the student correctly states and checks only one condition, *OR* if the student states both conditions correctly but checks neither of them.

Incorrect (I) otherwise.

Question 3 (continued)

Section 2 [part (b), step 1] is scored as follows:

Essentially correct (E) if the student states a correct pair of hypotheses.

Note: The hypotheses may be stated using words or using transparent variable notation such as p_{CPR} and p_{CP}, even if the parameters are not defined.

Incorrect (I) otherwise.

Note: Hypotheses that clearly address sample proportions are incorrect. It must be clear that the hypotheses are not about the sample proportions.

Section 3 [part (b), steps 2 and 3] is scored as follows:

Essentially correct (E) if the student identifies a correct test and includes correct mechanics.

Note: The mechanics are considered correct if the student uses a pooled test procedure or an unpooled test procedure, as long as the correct value of z is calculated.

Incorrect (I) otherwise.

Note: If the student writes the formula for the unpooled version but gets the correct values for the test statistic and p-value for the pooled version, section 3 is scored as incorrect.

Section 4 [part (b), step 4] is scored as follows:

Essentially correct (E) if the student states a correct conclusion in the context of the problem.

Note: A correct conclusion in context must explicitly state that the cardiopump survival rate is significantly higher than the CPR survival rate.

Incorrect (I) otherwise.

Notes:
- If the p-value in section 3 is incorrect but the conclusion is consistent with the computed p-value, section 4 can be considered correct.
- In section 4 if both an α and a p-value are given together, the linkage between the p-value and the conclusion is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p-value or explaining how the conclusion follows from the p-value.
- If, instead of a one-sided test, a student correctly performs a two-sided test (chi-square test for homogeneity of proportions or a two-sided z test for comparing two proportions), the final score drops automatically by 1 point.

Question 3 (continued)

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (a) counts as ½ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, 1½ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.

3A1

3. A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.

(a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.

Let n_1 = the number of patients with a cardiopump,
n_2 = the number of patients with CPR,
p_1 = the proportion of patients that survived with a cardiopump,
and p_2 = the proportion of patients that survived with CPR.

Conditions are satisfied for a 2-proportion z test:

① It's stated that both samples are taken independently and randomly.
(Randomization process is used, such as using a toss to decide treatment assignation.

② $n_1p_1 = 754 \times (37/754) = 37 > 10$, $n_1(1-p_1) = 754 \times (1-37/754) = 717 > 10$
$n_2p_2 = 746 \times (15/746) = 15 > 10$, $n_2(1-p_2) = 746 \times (1-15/746) = 731 > 10$

(b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

Using the definations of n_1, n_2, p_1 and p_2 stated in (a).

Step 1: We're interested in testing
$H_0: p_1 \leq p_2$ (The survival rate for patients with a cardiopump is not significantly higher than that for patients with CPR)

$H_A: p_1 > p_2$ (The survival ~~s~~ rate for patients with a cardiopump is significantly higher than that for patients with CPR)

Step 2: We'll use a 2-proportion Z test.
Conditions for such a test are satisfied, as stated in (a).

GO ON TO THE NEXT PAGE.

If you need more room for your work for part (b), use the space below.

Step 3: Compute the test statistic as

$$z = \frac{p_1 - p_2}{\sqrt{\hat{p}(1-\hat{p})(\frac{1}{n_1}+\frac{1}{n_2})}}, \text{ where } \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}.$$

$$\hat{p} = \frac{37+15}{754+746} = 0.03467$$

$$z = \frac{37/754 - 15/746}{\sqrt{0.03467 \times (1-0.03467) \times (\frac{1}{754}+\frac{1}{746})}} = \cancel{7.323}\ 3.066$$

$\rightarrow P(z > 3.066) = 0.0011$

Step 4: With a P-value this small, we have enough (actually pretty strong) evidence to reject H_0. There is enough evidence to claim that the survival rate for patients with a cardiopump is significantly higher than that for patients with CPR.

GO ON TO THE NEXT PAGE.

3B1

3. A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.

$p_1 = 0.04907$ $\quad p_2 = 0.020107$

(a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.

To compare the effectiveness of CPR & cardiopump, a two proportion z-test is to be performed.

The conditions for inference:

1. Randomly assigned treatments: the coin toss is random ~~treatment~~ assignment to the even-numbered or odd-numbered day of heart attack. ✓

2. $n_1 p_1 = (754)\left(\frac{37}{754}\right) = 37$, $n_2 p_2 = 746\left(\frac{15}{746}\right) = 15$, $n_1(1-p_1) = 754 \frac{717}{754} = 717$, $n_2(1-p_2) = 746\left(\frac{731}{746}\right) = 731$

These values are all greater than 5. condition met ✓

(b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

A z-test is to be performed to test the significance in the difference of proportions of survival rate for patients treated with a cardiopump and that of CPR.

(x= cardiopump, y= CPR)

H_0: if all the patients were to be assigned the cardio pump, the results of the survival rate would be exactly the same as the results if all patients were to be assigned CPR.

$p_x = p_y$

H_a: if all the patients were to be assigned the cardio pump, the survival rate of those patients will be greater than the survival rate if all patients were to be assigned CPR.

$p_x > p_y$

GO ON TO THE NEXT PAGE.

If you need more room for your work for part (b), use the space below.

$$z\text{-statistic} = \frac{\frac{37}{754} - \frac{15}{746}}{\sqrt{\frac{\frac{37}{754}\left(1-\frac{37}{754}\right)}{754} + \frac{\frac{15}{746}\left(1-\frac{15}{746}\right)}{746}}} = 3.066$$

p-valve: 0.00108

With 95% significance level, since the p-valve ≤ level of significance 0.05, the null hypothesis that if all patients were to be assigned the cardiopump, the results of survival rate would be the same if all patients were to be assigned CPR can be rejected. There is evidence to show that the difference of the survival rate is due to something other than pure chance.

The survival rate for patients treated with a cardiopump is statistically significantly higher than the survival rate for patients treated with CPR

GO ON TO THE NEXT PAGE.

3C1

3. A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.

(a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.

$754 = n_1 \equiv$ number treated with cardiopump

$746 = n_2 =$ number treated with CPR

$\frac{37}{754} = p_1 =$ proportion of survived, tr. with cardiopump

$\frac{15}{746} = p_2 =$ — , — , — / — . CPR

$n_1 \cdot p_1 = 37 > 10 \qquad n_1(1-p_1) = 717 > 10$

$n_2 \cdot p_2 = 15 > 10 \qquad n_2(1-p_2) = 731 > 0$

(b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

H_0 no difference in the proportion of survived between treatments

H_a proportion of siervived with cardiopump is higher

We conduct a two sample Z test or differance of proportions

Using calculator:

$Z_{stat} = 3.066 \qquad \alpha = 0.05$

p-value = 0.0108

Assuming 5% significance level, we reject the H_0 as p-value < α (1% < 5%)

GO ON TO THE NEXT PAGE.

If you need more room for your work for part (b), use the space below.

That means, that proportion of survived, treated with cardiopump is higher, and this method can be used.

GO ON TO THE NEXT PAGE.

Question 3

Sample: 3A
Score: 4

The only weak part of this response is the statement in part (a) that "both samples are taken independently and randomly." For an experiment, the randomness required is that the treatments are randomly assigned to the available subjects. In this study the available subjects apparently were all people who had heart attacks in the participating cities during the time of the study, so the two samples are not independent and are not random samples from a larger population to which the conclusion would apply. Nevertheless, the student's following statement ("[r]andomization process is used, such as using a toss to decide treatment assignation") conveys the right idea—that the coin flip should be equivalent to randomly assigning available subjects to the two treatments—so section 1 [part (a)] was scored as essentially correct. The hypotheses, test identification, mechanics, and conclusion steps in part (b) are well done, with sufficient work shown. Thus section 2 [part (b), step 1], section 3 [part (b), steps 2 and 3], and section 4 [part (b), step 4] were each scored as essentially correct. This complete response successfully and concisely accomplishes all four parts of a test of significance of the difference of two proportions and received a score of 4.

Sample: 3B
Score: 3

This response is very successful in stating and checking the conditions in part (a). It also very competently states the hypotheses and conclusion that are most appropriate for this experiment in part (b). The randomization, hypotheses, and conclusion are said to apply only to the available subjects: if all patients had been assigned the cardiopump, the survival rate would be higher than if all patients had been assigned CPR. Thus section 1 was scored as essentially correct, as were sections 2 and 4. However, the mechanics step in section 3 was scored as incorrect. The formula given for the standard error does not include a pooled estimate of the common proportion, yet the z-statistic of "3.066," presumably from the calculator, is the value obtained with a pooled estimate. Because sections 1, 2, and 4 were essentially correct, this substantial response received a score of 3.

Sample: 3C
Score: 2

Because there is no mention of random assignment of treatments to subjects, but the check that the sample sizes are sufficient is acceptable, section 1 of this response was scored as partially correct. In part (b) the hypotheses appear to be about the difference in the proportion of patients who "survived" (past tense) between the two treatments. Consequently, section 2—the hypotheses step—was scored as incorrect. Sections 3 and 4—the mechanics and conclusion steps—were scored as essentially correct, although the final sentence is not clear. Because section 1 was partially correct, section 2 was incorrect, section 3 was essentially correct, and section 4 was essentially correct, this developing response received a score of 2.