Question 2

Intent of Question

The primary goal of this question was to assess students' ability to evaluate conditional probabilities as they relate to diagnostic testing.

Solution

Part (a):

The estimated probability of a positive ELISA if the blood sample does not have HIV present is

$\frac{37}{500}$ *OR* $\frac{37}{500} = 0.074$

Part (b):

A total of 489 + 37 = 526 blood samples resulted in a positive ELISA. Of these, 489 samples actually contained HIV. Therefore the proportion of samples that resulted in a positive ELISA that actually contained HIV is

$\frac{489}{526}$ *OR* $\frac{489}{526} \approx 0.9297$

Part (c):

From part (a), the probability that the ELISA will be positive, given that the blood sample does not actually have HIV present, is 0.074. Thus, the probability of a negative ELISA, given that the blood sample does not actually have HIV present, is $1 - 0.074 = 0.926$.

P(new blood sample that does not contain HIV will be subjected to the more expensive test)

= *P*(*1st ELISA positive* and *2nd ELISA positive* <u>OR</u> *1st ELISA positive* and *2nd ELISA negative* and *3rd ELISA positive* | *HIV not present in blood*)
= *P*(*1st ELISA positive* and *2nd ELISA positive* | *HIV not present in blood*)
+ *P*(*1st ELISA positive* and *2nd ELISA negative* and *3rd ELISA positive* | *HIV not present in blood*)
= (0.074)(0.074) + (0.074)(0.926)(0.074)
= 0.005476 + 0.005070776
= 0.010546776
≈ 0.0105

OR

Question 2 (continued)

P(new blood sample that does not contain HIV will be subjected to the more expensive test)

$$
\begin{aligned}
&= P(\textit{1st ELISA positive and not both the 2nd and 3rd are negative}) \\
&= (0.074)(1 - 0.926^2) \\
&= (0.074)(0.142524) \\
&= 0.010546776 \\
&\approx 0.0105
\end{aligned}
$$

<u>Scoring</u>

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the student writes the correct fraction for the estimated probability.

Partially correct (P) if the decimal answer of 0.074 is given with no justification.

Incorrect (I) otherwise.

Part (b) is scored as follows:

Essentially correct (E) if the student writes the correct fraction for the proportion or gives a decimal approximation with justification.

Partially correct (P) if the student writes the wrong fraction but either correctly selects 489 as the numerator or correctly computes $489 + 37 = 526$ as the denominator.

Incorrect (I) otherwise.

Part (c) is scored as follows:

Essentially correct (E) if the student computes the correct probability, showing work.

Partially correct (P) if method is equivalent to P(*1st ELISA is positive*) · P(*at least one of two subsequent ELISAs is positive*) $= (0.074)(0.1425)$, except that one of the factors is incorrect.

OR

The student correctly computes P(*at least one of two subsequent ELISAs is positive*) = 0.1425, failing to include the first factor (0.074).

OR

The student correctly computes the probability of getting at least two positive ELISAs by testing negative blood three times:

$$\binom{3}{2}(0.074)^2(0.926) + \binom{3}{3}(0.074)^3 = 3(0.074)^2(0.926) + (0.074)^3 \approx 0.015617552$$

Incorrect (I) otherwise.

Question 2 (continued)

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and no parts partially correct

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and no parts partially correct

OR

No parts essentially correct and two parts partially correct

2. The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in a blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in a blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.

As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

(a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.

$P(\text{positive} \mid \text{does not contain HIV}) \approx \frac{37}{500} = 0.074.$

The probability that a new blood sample falsely tests positive for HIV using ELISA is approximately 0.074.

(b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?

There were a total of 489+37 = 526 positives. 489 of those samples truly had HIV. Thus, the ~~probability that~~ proportion of positive blood samples that truly had HIV is 489/526 = ~~0.93~~ 0.930

GO ON TO THE NEXT PAGE.

If you need more room for your work in part (b), use the space below.

(c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

The probability that the sample falsely tests positive the first time is approximately 0.074 as calculated in part (a).

The probability that the blood sample will yield at least one more false positive in the ELISA tests is the complement of getting 0 positives on the extra two tests. Assuming independence:

$1 - P(0\text{ positives}) = 1 - \left(1 - \frac{37}{500}\right)^2 = 1 - (0.926)^2 = 0.142524 = P(\text{at least one positive})$

Therefore, the probability of having a blood sample that does not contain HIV be subjected to the expensive test is approximately $(0.074)(0.142524) \approx 0.011$

GO ON TO THE NEXT PAGE.

2. The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in a blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in a blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.

As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

(a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.

note: p: positive n: negative y: HIV m: no HIV.

under the condition given by the question, we can conclude:

P (p | y) = 0.996 (given that one contains HIV, test shows positive)

P (n | y) = 0.004 (given that one contains HIV, test shows negative results)

$\frac{37}{500} = 0.074$ P (p | m) = 0.074 (given that one does not contain HIV, test shows positive results)

P (n | m) = 0.926 (given that one does not contain HIV, test shows negative results)

P (p | m) = 0.074 is what the question asks for, and the probability 0.074

(b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?

	HIV	no HIV	
positive	489	37	526
negative	11	463	474
total	500	500	1000

among the 526 ~~peop'~~ blood samples which showed positive ELISA results, 489 actually contained HIV, so the proportion is $\frac{489}{526} = 0.9297$, the same as 92.97%

GO ON TO THE NEXT PAGE.

2B2

If you need more room for your work in part (b), use the space below.

(c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

Because the ELISA tests are independent, and only have two possible results: positive, negative, we can fits the three test into a binomial distribution; everytime, there ~~are~~ is $P(p|m) = 0.074$ that the test is positive for blood sample with no HIV.

$$\binom{2}{1}(0.074)^1(0.926)^1 + \binom{2}{2}(0.074)^2(0.926)^0$$

$$= 0.137048 + 5.476 \times 10^{-3} = 0.142524$$

So there is 14.25% possibility the blood sample with no HIV would be sent to a more expensive experiment.

GO ON TO THE NEXT PAGE.

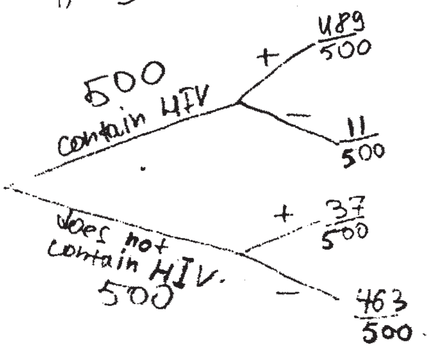
2C1

2. The ELISA tests whether a patient has contracted HIV. The ELISA is said to be positive if it indicates that HIV is present in a blood sample, and the ELISA is said to be negative if it does not indicate that HIV is present in a blood sample. Instead of directly measuring the presence of HIV, the ELISA measures levels of antibodies in the blood that should be elevated if HIV is present. Because of variability in antibody levels among human patients, the ELISA does not always indicate the correct result.

As part of a training program, staff at a testing lab applied the ELISA to 500 blood samples known to contain HIV. The ELISA was positive for 489 of those blood samples and negative for the other 11 samples. As part of the same training program, the staff also applied the ELISA to 500 other blood samples known to not contain HIV. The ELISA was positive for 37 of those blood samples and negative for the other 463 samples.

(a) When a new blood sample arrives at the lab, it will be tested to determine whether HIV is present. Using the data from the training program, estimate the probability that the ELISA would be positive when it is applied to a blood sample that does not contain HIV.

1) Show the situation on the tree:



2) let Pr(NO HIV) - the probability that the test does not contain HIV
Pr(P) - the probability that the test is positive
Pr(N) - the probability that the test is negative
Pr(HIV) - the probability that the test contains HIV

Therefore, we need to find the probability:

$$Pr(P \mid NO\,HIV) = \frac{Pr(P \cap NO\,HIV)}{Pr(NO\,HIV)} = \frac{\frac{37}{500}}{0,5} = 0,148 = 14,8\%$$

(b) Among the blood samples examined in the training program that provided positive ELISA results for HIV, what proportion actually contained HIV?

$$Pr(HIV \mid P) = \frac{Pr(HIV \cap P)}{Pr(P)} = \frac{\frac{1}{2} \cdot \frac{489}{500}}{\frac{1}{2} \cdot \frac{489}{500} + \frac{1}{2} \cdot \frac{37}{500}} = 0,9297 \approx 93\%$$

GO ON TO THE NEXT PAGE.

If you need more room for your work in part (b), use the space below.

(c) When a blood sample yields a positive ELISA result, two more ELISAs are performed on the same blood sample. If at least one of the two additional ELISAs is positive, the blood sample is subjected to a more expensive and more accurate test to make a definitive determination of whether HIV is present in the sample. Repeated ELISAs on the same sample are generally assumed to be independent. Under the assumption of independence, what is the probability that a new blood sample that comes into the lab will be subjected to the more expensive test if that sample does not contain HIV?

$Pr(P) = \frac{1}{2} \frac{489}{500} + \frac{1}{2} \cdot \frac{37}{500} = 0,526.$

A new sample will be subjected to te more expensive test if The ~~f~~test that does not contain HIV will be positive at least in one additional test, that is:

$Pr(P \mid NOHIV) = 0,148.$

will be subjected ~~with~~ te probability:

$Pr(\text{Subjected}) = 0,148 \cdot 0,148 \cdot 0,148 + 0,148 \; 0,148 \cdot 0,852 = 0,022 = 2,2\%$

GO ON TO THE NEXT PAGE.

Question 2

Sample: 2A
Score: 4

Each part of this question is clearly, concisely, and correctly answered. The student recognizes that the proportions in parts (a) and (b) can be found easily from the information given in the stem of the problem, without the use of complicated formulas. The method used in part (c) is clearly described. Using the result from part (a), the student correctly computes the probability of at least one additional positive test after the first one by finding the complement of the probability that both tests are negative. The two probabilities, 0.074 and 0.1425, are then correctly multiplied to find the probability that the first test is positive and at least one of two subsequent tests is positive. Parts (a), (b), and (c) were all scored as essentially correct. Consequently, this complete response received a score of 4.

Sample: 2B
Score: 3

Parts (a) and (b) of this response were scored as essentially correct. A table such as the one in part (b) makes it easy to understand which counts are involved in computing the proportions requested in parts (a) and (b). In part (c) a correct computation is given for the probability of getting at least one positive test out of the two additional tests of a blood sample that does not contain HIV. However, this probability, 0.1425, should be multiplied by the probability that the first test is positive, 0.074, to find the probability that the first test is positive and at least one of two subsequent tests is positive. Thus part (c) was scored as partially correct. Because part (a) was essentially correct, part (b) was essentially correct, and part (c) was partially correct, this substantial response received a score of 3.

Sample: 2C
Score: 2

In part (a) of this response the tree diagram organizes the counts involved but is used incorrectly to compute the probability that the ELISA test would be positive when it is applied to a blood sample that does not contain HIV. Thus part (a) was scored as incorrect. The formula in part (b) correctly uses the information from the tree diagram and was scored as essentially correct. In part (c) the computations are done using the answer from part (a)—as they should be. The fact that this part (a) answer is incorrect was ignored in the scoring of part (c), as the student had already been penalized for it. The probabilities of two of the three paths to a more expensive test are correctly computed and added, but the third path is omitted: the probability that the first test is positive, the second is negative, and the third is positive. Thus part (c) was scored as partially correct. Because part (a) was incorrect, part (b) was essentially correct, and part (c) was partially correct, this developing response received a score of 2.