General Notes About 2009 AP Physics Scoring Guidelines

1. The solutions contain the most common method of solving the free-response questions and the allocation of points for this solution. Some also contain a common alternate solution. Other methods of solution also receive appropriate credit for correct work.

2. Generally, double penalty for errors is avoided. For example, if an incorrect answer to part (a) is correctly substituted into an otherwise correct solution to part (b), full credit will usually be awarded. One exception to this may be cases when the numerical answer to a later part should be easily recognized as wrong, e.g., a speed faster than the speed of light in vacuum.

3. Implicit statements of concepts normally receive credit. For example, if use of the equation expressing a particular concept is worth one point and a student's solution contains the application of that equation to the problem, but the student does not write the basic equation, the point is still awarded. However, when students are asked to derive an expression it is normally expected that they will begin by writing one or more fundamental equations, such as those given on the AP Physics Exam equation sheet. For a description of the use of such terms as "derive" and "calculate" on the exams, and what is expected for each, see "The Free-Response Sections—Student Presentation" in the *AP Physics Course Description*.

4. The scoring guidelines typically show numerical results using the value $g = 9.8 \text{ m/s}^2$, but use of 10 m/s^2 is of course also acceptable. Solutions usually show numerical answers using both values when they are significantly different.

5. Strict rules regarding significant digits are usually not applied to numerical answers. However, in some cases answers containing too many digits may be penalized. In general, two to four significant digits are acceptable. Numerical answers that differ from the published answer due to differences in rounding throughout the question typically receive full credit. Exceptions to these guidelines usually occur when rounding makes a difference in obtaining a reasonable answer. For example, suppose a solution requires subtracting two numbers that should have five significant figures and that differ starting with the fourth digit (e.g., 20.295 and 20.278). Rounding to three digits will lose the accuracy required to determine the difference in the numbers, and some credit may be lost.

Question 1

15 points total **Distribution of points**

(a)

(i) 3 points

For indicating the field points radially inward — 1 point

$$E_r = -\frac{dV}{dr}$$

For correctly substituting the electric potential for $r < R$ into the equation for the electric field — 1 point

$$E_{inside} = -\frac{dV_{inside}}{dr} = -\frac{d}{dr}\left(\frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right]\right)$$

For correctly taking the derivative of the potential function to determine the magnitude of the electric field — 1 point

$$|E_{inside}| = \frac{Q_0}{4\pi\epsilon_0 R}\left[(3)(2)\left(\frac{r}{R}\right)\left(\frac{1}{R}\right)\right]$$

$$|E_{inside}| = \frac{6Q_0 r}{4\pi\epsilon_0 R^3}$$

(ii) 2 points

For indicating the field points radially outward — 1 point

$$E_r = -\frac{dV}{dr}$$

$$E_{outside} = -\frac{dV_{outside}}{dr} = -\frac{d}{dr}\left(\frac{Q_0}{4\pi\epsilon_0 r}\right)$$

For the correct answer — 1 point

$$|E_{outside}| = \frac{Q_0}{4\pi\epsilon_0 r^2}$$

Question 1 (continued)

Distribution of points

(b)

(i) 3 points

For using Gauss's law — 1 point

$$\oint \mathbf{E} \bullet d\mathbf{A} = \frac{Q_{enclosed}}{\epsilon_0}$$

For substituting the correct expression for the flux through a Gaussian sphere — 1 point

$$E 4\pi r^2 = \frac{Q_{enclosed,\ r<R}}{\epsilon_0}$$

$$E_{inside} = \frac{Q_{enclosed,\ r<R}}{4\pi\epsilon_0 r^2}$$

For substituting the appropriate expression for the electric field, consistent with the answer to part (a), including a minus sign — 1 point

$$-\frac{6Q_0 r}{4\pi\epsilon_0 R^3} = \frac{Q_{enclosed,\ r<R}}{4\pi\epsilon_0 r^2}$$

$$Q_{enclosed,\ r<R} = -\frac{6Q_0 r^3}{R^3}$$

(ii) 2 points

From Gauss's law as shown above

$$E = \frac{Q_{enclosed}}{4\pi\epsilon_0 r^2}$$

For correctly substituting the electric field from part (a) into the expression above — 1 point

$$E_{outside} = \frac{Q_0}{4\pi\epsilon_0 r^2} = \frac{Q_{enclosed,\ r>R}}{4\pi\epsilon_0 r^2}$$

$$Q_{enclosed,\ r>R} = Q_0$$

The field points outward, so the charge is positive.

For the correct enclosed charge for $r > R$ — 1 point

$$Q_{enclosed,\ r>R} = +Q_0$$

Question 1 (continued)

Distribution of points

(c) 2 points

There is charge residing on the surface of the sphere.

For indicating that the total enclosed charge equals the charge at the surface plus all the charge inside the sphere — 1 point

$Q_{enclosed,\ r>R} = Q_{surface} + Q_{enclosed,\ r<R}$ when $r=R$

$Q_{surface} = Q_{enclosed,\ r>R} - Q_{enclosed,\ r<R}$ when $r=R$

For substituting the correct value of $Q_{enclosed,\ r<R}$ when $r=R$ — 1 point

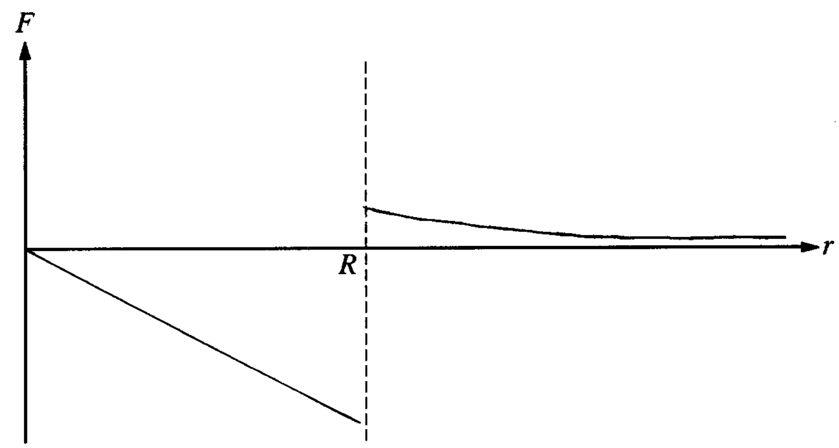
$$Q_{surface} = Q_0 - \left(-\frac{6Q_0R^3}{R^3}\right)$$

$$Q_{surface} = Q_0 - (-6Q_0)$$

$$Q_{surface} = 7Q_0$$

Note: The student could earn 1 point for a correct qualitative description of the charge configuration, even if the surface charge was not calculated.

(d) 3 points



For having the graph for $r < R$ consistent with the answer to part a (i) — 1 point

For having the graph for $r > R$ consistent with the answer to part a (ii), with a finite value at $r = R$ — 1 point

For having a step discontinuity at $r = R$ indicating the presence of charge at the surface (Having both graphs asymptotic to $r = R$ does not constitute the correct discontinuity.) — 1 point

PHYSICS C: ELECTRICITY AND MAGNETISM

SECTION II

Time—45 minutes

3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part, NOT in the green insert.

E&M. 1.

A spherically symmetric charge distribution has net positive charge Q_0 distributed within a radius of R. Its electric potential V as a function of the distance r from the center of the sphere is given by the following.

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \text{ for } r < R$$

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \text{ for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

✓ Radially inward ____ Radially outward

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left[\frac{-2Q_0}{4\pi\epsilon_0 R} + \frac{3Q_0 r^2}{4\pi\epsilon_0 R^3}\right]$$

$$= \frac{-3Q_0}{4\pi\epsilon_0 R^3}\cdot 2r = \boxed{\frac{-6Q_0 r}{4\pi\epsilon_0 R^3}}$$

$$= \boxed{\frac{-3Q_0 r}{2\pi\epsilon_0 R^3}}$$

As r increases V(r) increases
$\vec{E}$ lines are from high V to low V

ii. $r > R$

____ Radially inward ✓ Radially outward

$$E = \frac{dV}{dr} = -\frac{d}{dr}\left[\frac{Q_0}{4\pi\epsilon_0 r}\right] = \boxed{\frac{Q_0}{4\pi\epsilon_0 r^2}}$$

Since as r increases, V(r) decreases

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of *r*.

i. $r < R$

$$\oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0} = E(r)A = E(r)4\pi r^2$$

A is surface the area of the Gaussian surface

$$Q_{enc} = \varepsilon_0 E(r) 4\pi r^2$$

$$\Rightarrow Q_{enc} = \cancel{\varepsilon_0}\left(\frac{-6Q_0 r}{4\cancel{\pi}\cancel{\varepsilon_0}R^3}\right)4\cancel{\pi}r^2 = \boxed{\frac{-6Q_0 r^3}{R^3}}$$

ii. $r > R$

Same as before in that $Q_{enc} = \varepsilon_0 E(r) 4\pi r^2$

$$\Rightarrow Q_{enc} = \cancel{\varepsilon_0}\left(\frac{Q_0}{\cancel{4\pi}\cancel{\varepsilon_0}\cancel{r^2}}\right)\cancel{r^2}\cdot\cancel{4\pi} = \boxed{Q_0}$$

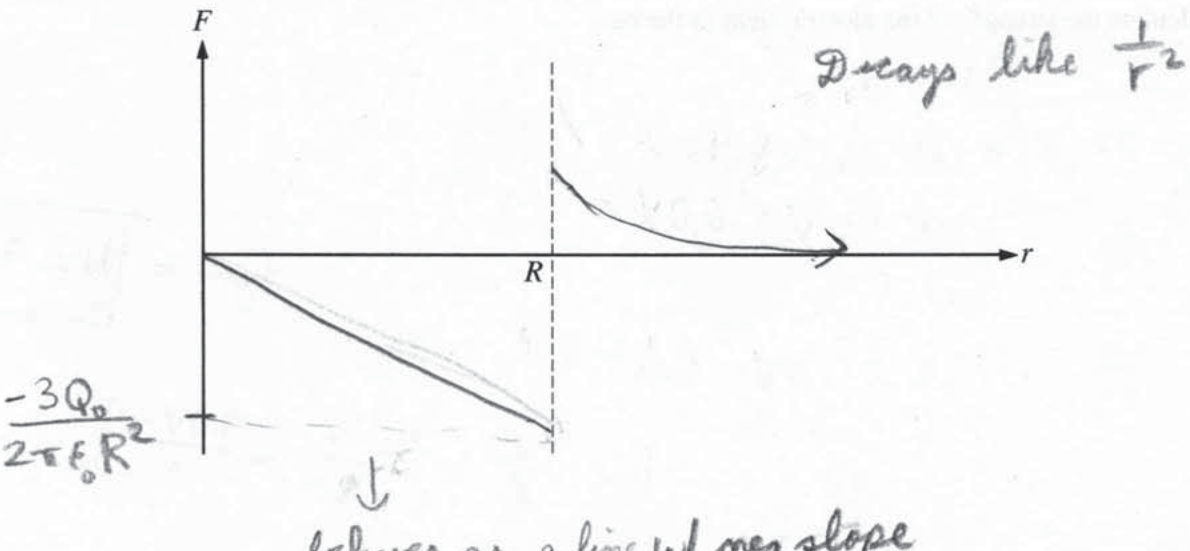
(c) Is there any charge on the surface of the sphere ($r = R$) ?

✓ Yes ____ No

If there is, determine the charge. In either case, explain your reasoning.

The charge on the surface is $7Q_0$.
When $r<R$, the enclosed charge is always negative w/ the Q_{enc} approaching $-6Q_0$ as $r \rightarrow R$.
When $r > R$, Q_{enc} is Q_0 meaning the surface has $7Q_0$ on it.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



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E&M. 1.

A spherically symmetric charge distribution has net positive charge Q_0 distributed within a radius of R. Its electric potential V as a function of the distance r from the center of the sphere is given by the following.

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 R}\left[-2 + 3\left(\frac{r}{R}\right)^2\right] \text{ for } r < R$$

$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \text{ for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

✓ Radially inward ____ Radially outward

$$E(r) = \frac{-dV(r)}{dr} = \frac{-Q_0}{4\pi\varepsilon_0 R}\left(0 + \frac{d}{dr}\left(\frac{3r^2}{R^2}\right)\right) = \frac{-Q_0}{4\pi\varepsilon_0 R}\left(\frac{6r}{R^2}\right)$$

$$\boxed{E(r) = \cancel{3Q_0}\ \frac{-3Q}{2\pi\varepsilon_0 R^3}r}$$

ii. $r > R$

____ Radially inward ✓ Radially outward

$$E(r) = \frac{-dV(r)}{dr} = -\frac{Q_0}{4\pi\varepsilon_0}\left(\frac{d}{dr}\left(\frac{1}{r}\right)\right) = \frac{-Q_0}{4\pi\varepsilon_0}\left(-\frac{1}{r^2}\right)$$

$$\boxed{E(r) = \frac{Q_0}{4\pi\varepsilon_0}\left(\frac{1}{r^2}\right)}$$

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i. $r < R$

Take a Gaussian surface (sphere) with radius r, $r < R$:

$\int E \cdot dA = E \cdot A = \frac{q_{enc}}{\varepsilon_0}$

$E = \frac{-3Q_0}{2\pi\varepsilon_0 R^3} r$, $A = 4\pi r^2$ Therefore, $\boxed{q_{enc} = \frac{-6Q_0 r^3}{R^3}}$

ii. $r > R$

Take a Gaussian sphere of radius r, $r > R$

$\int E \cdot dA = E \cdot A = \frac{q_{enc}}{\varepsilon_0}$

$E = \frac{Q_0}{4\pi\varepsilon_0}\left(\frac{1}{r^2}\right)$, $A = 4\pi r^2$ Therefore, $\boxed{q_{enc} = Q_0}$

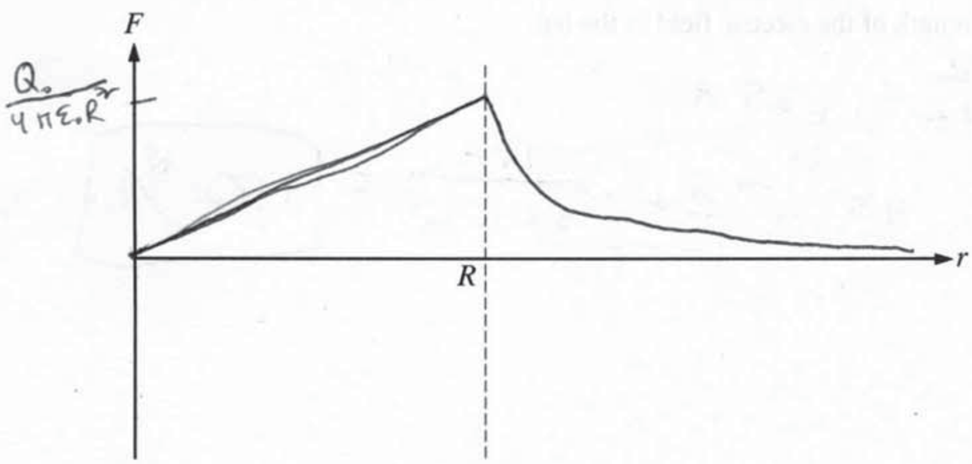
(c) Is there any charge on the surface of the sphere ($r = R$) ?

____ Yes ✓ No

If there is, determine the charge. In either case, explain your reasoning.

There is charge withing the sphere given by $q_{enc} = \frac{-6Qr^3}{R^3}$. Therefore, it is not a conductor and the charge is distributed throught the region not just on the surface.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



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$$V(r) = \frac{Q_0}{4\pi\epsilon_0 r} \text{ for } r > R$$

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field $E(r)$ and derive an expression for its magnitude.

i. $r < R$

____ Radially inward ✓ Radially outward

$$E = -\frac{dV}{dr} = -V'(r) = -\frac{dV}{dr}\left(\frac{-2Q_0}{4\pi\varepsilon_0 R} + \frac{3Q_0}{4\pi\varepsilon_0 R^3}r^2\right)$$

$$= -\frac{3Q_0}{4\pi\varepsilon_0 R^3}2r = \frac{6Q_0 r}{4\pi\varepsilon_0 R^3} = \boxed{-\frac{3Q_0 r}{2\pi\varepsilon_0 R^3}}$$

ii. $r > R$

____ Radially inward ✓ Radially outward

$$E = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{Q_0}{4\pi\varepsilon_0 r}\right) = -\frac{d}{dr}\left(\frac{Q_0 r^{-1}}{4\pi\varepsilon_0}\right)$$

$$= -\left(\frac{-Q_0 r^{-2}}{4\pi\varepsilon_0}\right) = \boxed{\frac{Q_0}{4\pi\varepsilon_0 r^2}}$$

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of r.

i. $r < R$

$$\rho = \frac{Q_0}{\frac{4}{3}\pi R^3}$$

$$\boxed{Q = Q_0 \frac{r^3}{R^3}}$$

ii. $r > R$

$$\boxed{Q_0}$$

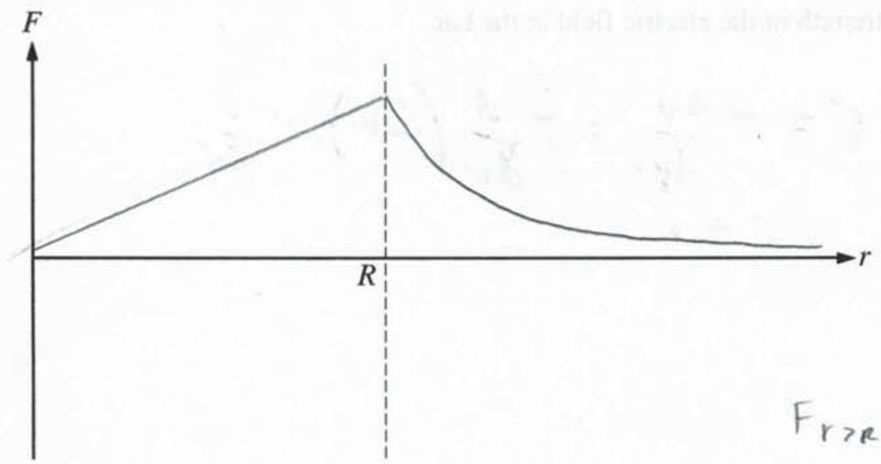
(c) Is there any charge on the surface of the sphere ($r = R$) ?

____ Yes ✓ No

If there is, determine the charge. In either case, explain your reasoning.

Since the charge is evenly distributed throughout the sphere, we can assume that the sphere is not made of a conductor material. A conductor would have all of its charge on the surface, but in this case, the non-conductor sphere will have no charge on its surface.

(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions $r < R$ and $r > R$. Assume that a force directed radially outward is positive.



$$F_{r<R} = k\frac{q_1 q_2}{r^2}$$

$$= k\frac{Q_0 \frac{r^3}{R^3} q}{r^2}$$

$$= \frac{kQ_0 q\, r^{\cancel{3}}}{R^3 \cancel{r^2}}$$

$$= \frac{kQ_0 q\, r}{R^3}$$

$$F_{r>R} = \frac{kQ_0 q}{r^2}$$

Question 1

Overview

Many of the major electrostatic concepts were tested in this question, which begins with a description of an unknown spherical distribution of charge and a given relationship for the electric potential of the charge distribution inside and outside the sphere.

In part (a) students were asked to derive the functions for the electric field, which tested their understanding of the differential relationship $E = -dV/dr$ between a spherically symmetric electric potential and electric field. Since the given potential function was not a simple function, students' ability to correctly take a derivative was also tested. In part (b) which assessed students' understanding of Gauss's law, they were asked to use the electric field functions from part (a) to determine the charge distribution.

In part (c) students were asked about a possible surface charge on the sphere. (The overall charge distribution consisted of negative charge distributed within the sphere, plus positive charge placed on the surface of the sphere.) Students who correctly answered part (c) demonstrated a very good understanding of Gauss's law. In part (d) students were asked to graph the resulting electric force on a positive test charge. This assessed their understanding of the relationship between electrostatic force and field, as well as their visual understanding of the charge distribution. A correctly drawn graph would clearly show a discontinuity at the surface of the sphere, which allowed some students to uncover and resolve mathematical or conceptual mistakes that they may have made in earlier parts of the problem.

Sample: CE-1A
Score: 15

The solution in this response is nicely done. The student shows all steps and gives a nice explanation in part (c). The graph in part (d) has a well-proportioned discontinuity.

Sample: CE-1B
Score: 11

This solution earned the full 5 points in part (a). Part (b) is also nicely done and earned the maximum of 5 points as well. No points were earned in part (c). The graph in part (d) earned only 1 point since the $r < R$ section is not negative, as it should be to be consistent with the "inward" direction in part (a)(i). Also, it does not show the discontinuity at $r = R$.

Sample: CE-1C
Score: 7

The response earned only 2 points for part (a)(i) since "Radially outward" is checked instead of "Radially inward." Part (a)(ii) earned the full 2 points. Part (b) earned only 1 point for the correct answer in (ii). No points were earned in part (c). The graph in part (d) earned 2 points for each of the segments being consistent with part (a), but 1 point was lost for not showing a discontinuity.