<table>
<thead>
<tr>
<th>UNIT SYMBOLS</th>
<th>FACTOR</th>
<th>PREFIX</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>meter, m</td>
<td>10^9</td>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>kilogram, kg</td>
<td>10^6</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>second, s</td>
<td>10^3</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>ampere, A</td>
<td>10^{-2}</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>pascal, Pa</td>
<td>10^{-3}</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>joule, J</td>
<td>10^{-6}</td>
<td>micro</td>
<td>µ</td>
</tr>
<tr>
<td>henry, H</td>
<td>10^{-9}</td>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>farad, F</td>
<td>10^{-12}</td>
<td>pico</td>
<td>p</td>
</tr>
</tbody>
</table>

### VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

<table>
<thead>
<tr>
<th>θ</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>sinθ</td>
<td>0</td>
<td>1/2</td>
<td>3/5</td>
<td>5/3</td>
<td>1</td>
</tr>
<tr>
<td>cosθ</td>
<td>1</td>
<td>√3/2</td>
<td>4/5</td>
<td>3/2</td>
<td>1/2</td>
</tr>
<tr>
<td>tanθ</td>
<td>0</td>
<td>√3/3</td>
<td>3/4</td>
<td>1</td>
<td>4/3</td>
</tr>
</tbody>
</table>

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.
### MECHANICS

- \( v = v_0 + at \)  
- \( x = x_0 + v_0 t + \frac{1}{2} a t^2 \)  
- \( v^2 = v_0^2 + 2a(x - x_0) \)  
- \( \sum F = F_{\text{net}} = ma \)  
- \( F = \frac{dp}{dt} \)  
- \( J = \int F \, dt = \Delta p \)  
- \( p = mv \)  
- \( F_{\text{fric}} \leq \mu N \)  
- \( W = \int F \cdot dr \)  
- \( K = \frac{1}{2} mv^2 \)  
- \( W = \text{work done on a system} \)  
- \( P = \frac{dW}{dt} \)  
- \( P = F \cdot v \)  
- \( \Delta U_\phi = mgh \)  
- \( a_c = \frac{v^2}{r} = \omega^2 r \)  
- \( \tau = \mathbf{r} \times \mathbf{F} \)  
- \( \Sigma \tau = \tau_{\text{net}} = I \alpha \)  
- \( L = \int r^2 \, dm = \Sigma mr^2 \)  
- \( r_{\text{cm}} = \Sigma mr/\Sigma m \)  
- \( \nu = r \omega \)  
- \( L = \mathbf{r} \times \mathbf{p} = I \omega \)  
- \( K = \frac{1}{2} I \omega^2 \)  
- \( \omega = \omega_0 + at \)  
- \( U_G = -\frac{Gm_1m_2}{r} \)  
- \( \theta = \theta_0 + \omega_0 t + \frac{1}{2} a t^2 \)

### ELECTRICITY AND MAGNETISM

- \( F = \frac{1}{4\pi \epsilon_0} \frac{q_1q_2}{r^2} \)  
- \( E = \frac{F}{q} \)  
- \( \oint \mathbf{E} \cdot dA = \frac{Q}{\epsilon_0} \)  
- \( E = -\frac{dV}{dr} \)  
- \( V = \frac{1}{4\pi \epsilon_0} \sum_i \frac{q_i}{r_i} \)  
- \( U_E = qV = \frac{1}{4\pi \epsilon_0} \frac{q_1q_2}{r} \)  
- \( C = \frac{Q}{V} \)  
- \( C = \frac{\kappa \epsilon_0 A}{d} \)  
- \( C_p = \sum_i C_i \)  
- \( \frac{1}{C_s} = \sum_i \frac{1}{C_i} \)  
- \( I = \frac{dQ}{dt} \)  
- \( U_c = \frac{1}{2} QV = \frac{1}{2} CV^2 \)  
- \( \oint \mathbf{B} \cdot d\ell = \mu_0 I \)  
- \( d\mathbf{B} = \frac{\mu_0 I \, d\ell \times \mathbf{r}}{4\pi r^3} \)  
- \( \mathbf{F} = \int l \, d\ell \times \mathbf{B} \)  
- \( B_s = \mu_0 nI \)  
- \( \phi_m = \int \mathbf{B} \cdot dA \)  
- \( E = -\frac{d\phi_m}{dt} \)  
- \( \mathbf{E} = -L \frac{dl}{dt} \)  
- \( P = IV \)  
- \( U_L = \frac{1}{2} LI^2 \)
### GEOMETRY AND TRIGONOMETRY

- **Rectangle**
  \[ A = bh \]
  
- **Triangle**
  \[ A = \frac{1}{2}bh \]
  
- **Circle**
  \[ A = \pi r^2 \]
  \[ C = 2\pi r \]

- **Parallelepiped**
  \[ V = \ell wh \]

- **Cylinder**
  \[ V = \pi r^2 \ell \]
  \[ S = 2\pi r \ell + 2\pi r^2 \]

- **Sphere**
  \[ V = \frac{4}{3}\pi r^3 \]
  \[ S = 4\pi r^2 \]

- **Right Triangle**
  \[ a^2 + b^2 = c^2 \]
  \[ \sin \theta = \frac{a}{c} \]
  \[ \cos \theta = \frac{b}{c} \]
  \[ \tan \theta = \frac{a}{b} \]

### CALCULUS

\[
\frac{df}{dx} = \frac{df}{du} \frac{du}{dx}
\]

\[
\frac{d}{dx} \left( x^n \right) = nx^{n-1}
\]

\[
\frac{d}{dx} \left( e^x \right) = e^x
\]

\[
\frac{d}{dx} \left( \ln x \right) = \frac{1}{x}
\]

\[
\frac{d}{dx} \left( \sin x \right) = \cos x
\]

\[
\frac{d}{dx} \left( \cos x \right) = -\sin x
\]

\[
\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1
\]

\[
\int e^x \, dx = e^x
\]

\[
\int \frac{dx}{x} = \ln |x|
\]

\[
\int \sin x \, dx = -\cos x
\]
E&M. 1.

A spherically symmetric charge distribution has net positive charge \( Q_0 \) distributed within a radius of \( R \).

Its electric potential \( V \) as a function of the distance \( r \) from the center of the sphere is given by the following.

\[
V(r) = \frac{Q_0}{4\pi \varepsilon_0 r} \quad \text{for } r > R
\]

\[
V(r) = \frac{Q_0}{4\pi \varepsilon_0 R} \left( -2 + 3 \left( \frac{r}{R} \right)^2 \right) \quad \text{for } r < R
\]

Express all algebraic answers in terms of the given quantities and fundamental constants.

(a) For the following regions, indicate the direction of the electric field \( E(r) \) and derive an expression for its magnitude.

i. \( r < R \)

\[ \text{_____ Radially inward} \quad \text{_____ Radially outward} \]

ii. \( r > R \)

\[ \text{_____ Radially inward} \quad \text{_____ Radially outward} \]

(b) For the following regions, derive an expression for the enclosed charge that generates the electric field in that region, expressed as a function of \( r \).

i. \( r < R \)

ii. \( r > R \)

(c) Is there any charge on the surface of the sphere \( (r = R) \) ?

\[ \text{_____ Yes} \quad \text{_____ No} \]

If there is, determine the charge. In either case, explain your reasoning.
(d) On the axes below, sketch a graph of the force that would act on a positive test charge in the regions \( r < R \) and \( r > R \). Assume that a force directed radially outward is positive.
E&M. 2.

A 9.0 V battery is connected to a rectangular bar of length 0.080 m, uniform cross-sectional area $5.0 \times 10^{-6}$ m$^2$, and resistivity $4.5 \times 10^{-4}$ $\Omega \cdot$m, as shown above. Electrons are the sole charge carriers in the bar. The wires have negligible resistance. The switch in the circuit is closed at time $t = 0$.

(a) Calculate the power delivered to the circuit by the battery.

(b) On the diagram below, indicate the direction of the electric field in the bar.

Explain your answer.

(c) Calculate the strength of the electric field in the bar.

A uniform magnetic field of magnitude 0.25 T perpendicular to the bar is added to the region around the bar, as shown below.

(d) Calculate the magnetic force on the bar.

(e) The electrons moving through the bar are initially deflected by the external magnetic field. On the diagram below, indicate the direction of the additional electric field that is created in the bar by the deflected electrons.

(f) The electrons eventually experience no deflection and move through the bar at an average speed of $3.5 \times 10^{-3}$ m/s. Calculate the strength of the additional electric field indicated in part (e).
E&M. 3.

A square conducting loop of side \( L \) contains two identical lightbulbs, 1 and 2, as shown above. There is a magnetic field directed into the page in the region inside the loop with magnitude as a function of time \( t \) given by 
\[
B(t) = at + b,
\]
where \( a \) and \( b \) are positive constants. The lightbulbs each have constant resistance \( R_0 \). Express all answers in terms of the given quantities and fundamental constants.

(a) Derive an expression for the magnitude of the emf generated in the loop.

(b)  
   i. Determine an expression for the current through bulb 2.
   ii. Indicate on the diagram above the direction of the current through bulb 2.

(c) Derive an expression for the power dissipated in bulb 1.

Another identical bulb 3 is now connected in parallel with bulb 2, but it is entirely outside the magnetic field, as shown below.

(d) How does the brightness of bulb 1 compare to what it was in the previous circuit?
   
   _____ Brighter       _____ Dimmer       _____ The same

   Justify your answer.
Now the portion of the circuit containing bulb 3 is removed, and a wire is added to connect the midpoints of the top and bottom of the original loop, as shown below.

(e) How does the brightness of bulb 1 compare to what it was in the first circuit?

_____ Brighter    _____ Dimmer    _____ The same

Justify your answer.