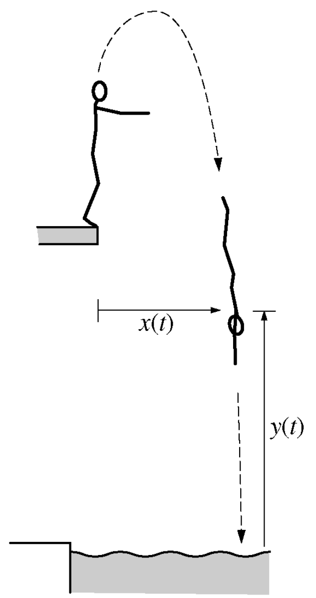
AP® CALCULUS BC
2009 SCORING GUIDELINES

Question 3

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.



Note: Figure not drawn to scale.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

(a) $\frac{dy}{dt} = 0$ only when $t = 0.36735$. Let $b = 0.36735$. The maximum vertical distance from the water surface to the diver's shoulders is $y(b) = 11.4 + \int_0^b \frac{dy}{dt}\,dt = 12.061$ meters. Alternatively, $y(t) = 11.4 + 3.6t - 4.9t^2$, so $y(b) = 12.061$ meters.	3 : { 1 : considers $\frac{dy}{dt} = 0$; 1 : integral or $y(t)$; 1 : answer }
(b) $y(A) = 11.4 + \int_0^A \frac{dy}{dt}\,dt = 11.4 + 3.6A - 4.9A^2 = 0$ when $A = 1.936$ seconds.	2 : { 1 : equation; 1 : answer }
(c) $\int_0^A \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = 12.946$ meters	2 : { 1 : integral; 1 : answer }
(d) At time A, $\frac{dy}{dx} = \left.\frac{dy/dt}{dx/dt}\right\|_{t=A} = -19.21913$. The angle between the path of the diver and the water is $\tan^{-1}(19.21913) = 1.518$ or 1.519.	2 : { 1 : $\frac{dy}{dx}$ at time A; 1 : answer }

Work for problem 3(a)

$3.6 - 9.8t = 0$

$t = 0.367$

$11.4 + \int_0^{0.367} 3.6 - 9.8t \, dt \approx \boxed{12.061 \text{ meters}}$

Work for problem 3(b)

$11.4 + \int_0^{A} 3.6 - 9.8t \, dt = 0$

$\int_0^{A} 3.6 - 9.8t \, dt = -11.4$

$A \approx \boxed{1.936 \text{ sec}}$

Continue problem 3 on page 9.

3A$_2$

Work for problem 3(c)

$$\int_0^{1.936} \sqrt{(0.8)^2+(3.6-9.8t)^2}\,dt \approx \boxed{12.946 \text{ meters}}$$

Work for problem 3(d)

$$\frac{dy}{dx} = \frac{3.6-9.8t}{0.8}$$

$$\left.\frac{dy}{dx}\right|_{t=1.936} \approx -19.219$$

$$\theta = |\arctan(-19.219)| \approx \boxed{1.519}$$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

 3B1

Work for problem 3(a)

$$\frac{dy}{dt} = 0 = 3.6 - 9.8t$$

$$t = .3673469388$$

$$11.4 + \int_0^{.3673469388} (3.6 - 9.8t)\,dt = 12.06122449 \text{ meters}$$

Do not write beyond this border.

Work for problem 3(b)

$$11.4 + \int_0^A (3.6 - 9.8t)\,dt = 0$$

$$11.4 + \Big|_0^A (3.6t - 4.9t^2) = 0$$

$$11.4 + 3.6A - 4.9A^2 = 0$$

$$A = 1.93626 \text{ seconds}$$

Continue problem 3 on page 9.

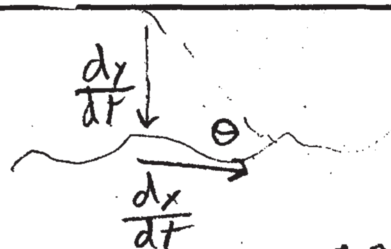
3B2

Do not write beyond this border.

Work for problem 3(c)

$12.06122449 + \int_0^{.3673469388} (3.6 - 9.8t)\,dt = 12.72245$ meters

Work for problem 3(d)



$\tan\theta = \dfrac{\frac{dy}{dt}(A)}{\frac{dx}{dt}(A)} = \dfrac{3.6 - 9.8A}{0.8} = -19.219185$

$\theta = \arctan -19.219185 = -1.5708$

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Do not write beyond this border.

3C1

Do not write beyond this border.

Work for problem 3(a)

$\frac{dy}{dt} / \frac{dx}{dt} = \frac{dy}{dx} = \frac{3.6 - 9.8t}{.8}$

max at $\frac{dy}{dt} = 3.6 - 9.8t = 0$

$\boxed{t = .367 \text{sec}}$

Work for problem 3(b)

enters water at $y=0$.

$\int \frac{dy}{dt} = 3.6t - 4.9t^2 + 11.4 = 0$

$\boxed{t = 1.93626}$ sec

Do not write beyond this border.

Continue problem 3 on page 9.

Do not write beyond this border.

Work for problem 3(c)

arc length $= \int_a^b \sqrt{(x'(t))^2 + y'(t)^2}\, dt$

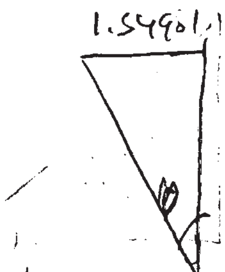
$\int_0^{1.93626} \sqrt{(.8)^2 + (3.6 - 9.8t)^2}\, dt = \boxed{12.9463 \text{ meters}} + 11.4 \text{ meters}$

$= \boxed{24.346 \text{ meters}}$

Work for problem 3(d)

at $t = 1.93626$ – when the diver enters the water – $\int \frac{dx}{dt} dt = .8t$ & $\int \frac{dy}{dt} = 3.6t - 4.9t^2 + 11.4$

at $t = 1.93626$, $x(t) = 1.54901$, $y(t) = 16.5333$



$\tan\theta = \frac{1.54901}{16.5333} = .093689$

$\tan^{-1}(.093689) =$ $\boxed{.0934 \text{ radians}}$

Do not write beyond this border.

END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

Question 3

Overview

This problem described the path of a diver's shoulders during a dive from a platform into a pool, using parametric functions $x(t)$ for the horizontal distance from the edge of the platform and $y(t)$ for the vertical distance from the water. It was stated that the diver's shoulders were 11.4 meters above the water at the start, and that $\frac{dx}{dt} = 0.8$ and $\frac{dy}{dt} = 3.6 - 9.8t$ for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water. Part (a) asked for the maximum height of the diver's shoulders above the water. Students needed to use the given derivative of y to determine when the diver's shoulders were highest and then to combine the initial height of the shoulders above the water with the appropriate integral to determine the maximum height. Part (b) asked for the time A that the diver's shoulders enter the water. This involved integrating to find the height of the shoulders above the water at time A and then solving for when this height was zero. Part (c) asked for the total distance the diver's shoulders traveled during the dive, which required an arclength calculation for the curve described by $x(t)$ and $y(t)$ from time 0 to the time A found in part (b). Part (d) asked for the acute angle between the path of the diver and the water's surface at time A. Students needed to find the slope of the path, $\frac{dy}{dx}$, at time A and then solve for the angle by realizing that its tangent matches the absolute value of this slope.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 6 points: 3 points in part (a), 2 points in part (b), no points in part (c), and 1 point in part (d). In parts (a) and (b) the student's work is correct. In part (c) the student's definite integral does not represent a calculation of arclength. In part (d) the student correctly evaluates the ratio $\frac{dx/dt}{dy/dt}$ at the value of A and earned the first point. The student presents a value for θ that is not in the interval $0 < \theta < \frac{\pi}{2}$ and did not earn the second point.

Sample: 3C
Score: 4

The student earned 4 points: 1 point in part (a), 2 points in part (b), 1 point in part (c), and no points in part (d). In part (a) the student considers $\frac{dy}{dt} = 0$ and earned the first point. In part (b) the student's work is correct. In part (c) the student earned the first point for setting up the correct definite integral for the calculation of arclength. The integral is evaluated correctly, but the student incorrectly adds the initial distance of the diver at $t = 0$. The answer point was not earned. In part (d) the student incorrectly calculates the ratio of distances $\frac{y(t)}{x(t)}$ at A instead of $\frac{dy}{dx}$ at A and was not eligible for any points.