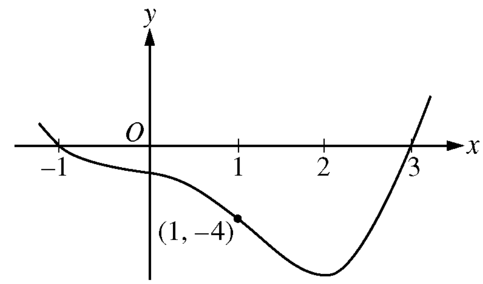
AP® CALCULUS BC
2009 SCORING GUIDELINES (Form B)

Question 5

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2$. The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2$. Let g be the function given by $g(x) = e^{f(x)}$.



Graph of f'

(a) Write an equation for the line tangent to the graph of g at $x = 1$.

(b) For $-1.2 < x < 3.2$, find all values of x at which g has a local maximum. Justify your answer.

(c) The second derivative of g is $g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$. Is $g''(-1)$ positive, negative, or zero? Justify your answer.

(d) Find the average rate of change of g', the derivative of g, over the interval $[1, 3]$.

(a) $g(1) = e^{f(1)} = e^2$ $g'(x) = e^{f(x)} f'(x)$, $g'(1) = e^{f(1)} f'(1) = -4e^2$ The tangent line is given by $y = e^2 - 4e^2(x - 1)$.	$3: \begin{cases} 1: g'(x) \\ 1: g(1) \text{ and } g'(1) \\ 1: \text{tangent line equation} \end{cases}$
(b) $g'(x) = e^{f(x)} f'(x)$ $e^{f(x)} > 0$ for all x So, g' changes from positive to negative only when f' changes from positive to negative. This occurs at $x = -1$ only. Thus, g has a local maximum at $x = -1$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(c) $g''(-1) = e^{f(-1)}\left[(f'(-1))^2 + f''(-1)\right]$ $e^{f(-1)} > 0$ and $f'(-1) = 0$ Since f' is decreasing on a neighborhood of -1, $f''(-1) < 0$. Therefore, $g''(-1) < 0$.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{justification} \end{cases}$
(d) $\dfrac{g'(3) - g'(1)}{3 - 1} = \dfrac{e^{f(3)} f'(3) - e^{f(1)} f'(1)}{2} = 2e^2$	$2: \begin{cases} 1: \text{difference quotient} \\ 1: \text{answer} \end{cases}$

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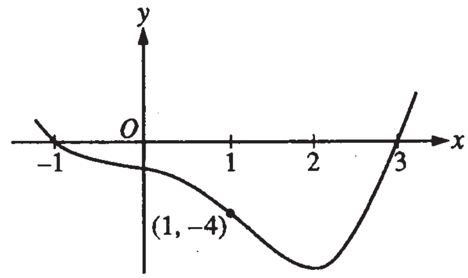

Graph of f'

Work for problem 5(a)

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^{2}$

$e^{2} = -4e^{2} + C$

$\Rightarrow C = 5e^{2}$

Equation of tangent line to g at $x = 1$:

$y = -4e^{2}x + 5e^{2}$ //

Work for problem 5(b)

g has a local maximum when g' changes sign from positive to negative.

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)}$ is always positive, $\therefore$ $g'(x)$ changes sign from positive to negative when $f'(x)$ does so.

$f'(x)$ changes sign from positive to negative at $x = -1$,

$\therefore$ g has a local maximum at $x = -1$. //

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Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x)\right]$

$e^{f(x)}$ is always positive

$(f'(-1))^2 = 0$

$f''(-1)$ is negative.

$\therefore g''(-1)$ is negative.

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = -4e^2$

$g'(3) = e^{f(3)} \cdot f'(3) = 0$

Average rate of change $= \dfrac{0 - (-4e^2)}{3 - 1}$

$= 2e^2$

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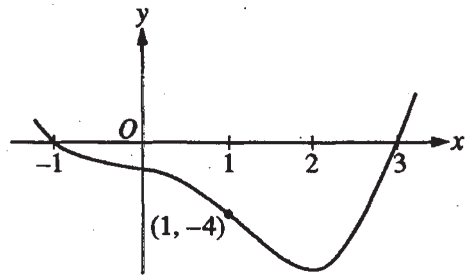

Graph of f'

Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{f(1)} \cdot f'(1) = e^2 \cdot -4 = -4e^2$

$g(1) = e^{f(1)} = e^2$

$\boxed{y - e^2 = -4e^2(x-1)}$

Work for problem 5(b)

$g'(x) = e^{f(x)} \cdot f'(x)$

$e^{f(x)} \Rightarrow$ always positive

g has a local maximum at $x = -1$ because $g'(x)$ changes from positive to negative.

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Work for problem 5(c)

$g''(x) = e^{f(x)}\left[(f'(x))^2 + f''(x) \right]$

$g''(-1)$ is negative, $e^{f(x)}$ is positive because any ~~e~~ raised to any number is positive, $f'(-1) = 0$ (given) and $f''(-1) < 0$ (from the graph), so $g''(-1)$ is a positive * (zero + negative) which comes out to be a negative value,

Work for problem 5(d)

average rate of change of $g' = \frac{1}{3-1} \int_1^3 g''(x)\,dx$

$= \frac{1}{3-1} \cdot g'(x)\Big|_1^3 = \frac{g'(3) - g'(1)}{2}$

$g'(3) = e^{f(3)} \cdot f'(3) = e^{f(3)} \cdot 0 = 0$

$g'(1) = -4e^2$ (from 5(a))

$= \frac{0 - (-4e^2)}{2} = \frac{4e^2}{2} = \boxed{2e^2}$

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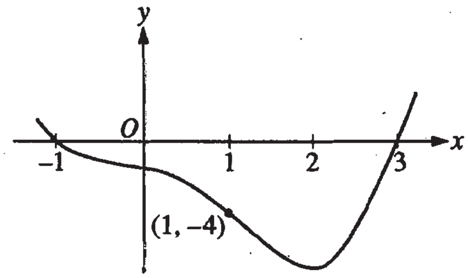

Graph of f'

Work for problem 5(a)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g'(1) = e^{2} \cdot (-4)$

$= -4e^{2}$

$(y - e) = -4e^{2}(x-1)$

Work for problem 5(b)

$f(x)$ has local max at $x=-1$

$g(x) = e^{f(x)}$

$\therefore g(x)$ has local max at $x=-1$

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Work for problem 5(c)

$f'(-1) = 0$

$f''(-1) < 0$ since $f'(x)$ is decreasing from $[-1/2, 2]$

since $f(1) = 2$ and $f(x)$ has only decreased from $f(-1)$ to $f(1)$, $f(-1) > 0$

$g''(x) = e^{f(x)}(0 + f''(x)$

$\therefore g''(-1) < 0$

Work for problem 5(d)

$g'(x) = e^{f(x)} \cdot f'(x)$

$g(1) = e^2 \cdot -4$

$= -4e^2$

$g(3) = 0$

avg rate of change $= \dfrac{g(3) - g(0)}{3 - 0}$

$= \dfrac{0 + 4e^2}{3}$

$= \dfrac{4e^2}{3}$

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Question 5

Sample: 5A
Score: 9

The student earned all 9 points. Note that in part (a) the student's first line earned the point for $g'(x)$. The student includes $g(1)$ implicitly in the second equation. In part (c) the justification is sufficient although the student does not explain why $f''(-1)$ is negative.

Sample: 5B
Score: 6

The student earned 6 points: 3 points in part (a), 1 point in part (b), 2 points in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the answer point, but the justification is insufficient. The student does not describe the sign change in g'. In part (c) the student's work is correct. In part (d) the student is not working with the correct difference quotient.

Sample: 5C
Score: 3

The student earned 3 points: 1 point in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student earned the point for $g'(x)$. The student does not have a value for $g(1)$. As a result, the second point was not earned, and the student was not eligible for the third point. In parts (b) and (c) the student earned the answer points. Both justifications are insufficient. In part (d) the student is not working with the correct difference quotient.