Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t)\,dt = 980$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$

The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$
$R(a) = 854.527$
$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

$3 : \begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t)\,dt = \int_1^2 (2 - t)R(t)\,dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980}w(2) = \frac{1}{980}\int_0^2 (2 - t)R(t)\,dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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Work for problem 2(a)

$\int_0^2 R(t)\,dt = \boxed{980 \text{ people}}$

Work for problem 2(b)

$\underline{R'(t)=0}$

$t=1.3629=A$

$\underline{\text{Endpoints}}$

$t=0$

$t=2$

R has an absolute maximum at $t=1.3629$ hours on $t\in[0,2]$ guaranteed by the EVT.

t	$R(t)$
0	0
A	854.5273
2	120

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Work for problem 2(c)

$\int_1^2 w'(t)\,dt = \boxed{387.5 \text{ hours}}$

Work for problem 2(d)

$\int_0^2 w'(t)\,dt \div \int_0^2 R(t)\,dt = \frac{760}{980} = \boxed{0.7755 \ \frac{\text{hours}}{\text{person}}}$

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Work for problem 2(a)

People $= \int_0^2 R(t)\,dt = 980$ people

There are approximately 980 people in the auditorium when the concert begins.

Work for problem 2(b)

$R(t) = 1380t^2 - 675t^3$

$R'(t) = 2760t - 2025t^2$

$0 = 2760t - 2025t^2$

$t = 0, 1.363$

$R'(t)$ − 0 + 0 −

-1 0 1 1.363 2

$R'(-1) = -4785$ $R'(2) = -2580$

$R'(1) = 735$

The rate at which people enter the auditorium is at a max at $t = 1.363$ hour b/c $R'(t) = 0$ at $t = 1.363$ hours & changes from + to −

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Work for problem 2(c)

$\int_a^b w'(t) = w(b) - w(a)$

$w'(t) = (2-t)R(t)$

$w(2) - w(1) = \int_1^2 (w'(t))\,dt$

$= \int_1^2 (2-t)(1380t^2 - 675t^3)\,dt$

$= 387.5$ hours

Total wait time is 387.5 hours according to the fundamental theorem of Calculus

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Work for problem 2(d)

Avg. wait time $= \frac{1}{b-a}\int_a^b w'(t)\,dt$

$= \frac{1}{2-1}\int_1^2 w'(t)\,dt$

$= \frac{1}{1}\int_1^2 w'(t)\,dt$

$= 387.5$

On average a person waits 387.5 hours in the auditorium for concert to begin

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Work for problem 2(a)

$$\int_0^2 1380t^2 - 675t^3 \, dt$$

$$1380 \cdot \frac{1}{3} t^3 - 675 \cdot \frac{1}{4} t^4$$

$$460t^3 - 168.75t^4$$

$$\left(460(2)^3 - 168.75(2)^4\right) - \left(460(0)^3 - 168.75(0)^4\right)$$

$980 - 0 = 980$ people are in the auditorium when the concert begins

Work for problem 2(b)

$$R(t) = 1380t^2 - 675t^3$$

$$R'(t) = 2760t - 2025t^2$$

$$2760t - 2025t^2 = 0$$

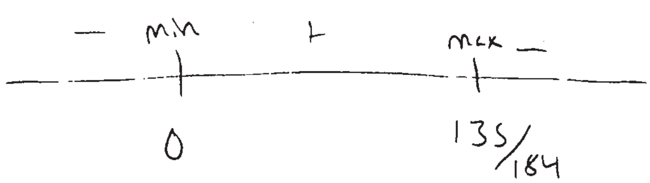
$$t(2760 - 2025t) = 0$$

$$t = 0, \ 135/184$$

$$2760 - 2025t = 0$$

$$-2025t = -2760$$

$$t = 135/184$$



The rate at which people enter the auditorium is at its maximum at time 135/184.

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Work for problem 2(c)

$w'(t) = (2-t)\,R(t)$

$w'(t) = (2-t)(1380t^2 - 675t^3)$

$\int_1^2 (2-t)(1380t^2 - 675t^3)\,dt =$ ______ hours

Work for problem 2(d)

$$\frac{\text{\_\_\_\_\_\_ hours = the total wait time}}{\text{980 people who are in the auditorium when the concert begins}}$$

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Question 2

Overview

This problem presented students with a polynomial $R(t) = 1380t^2 - 675t^3$ that modeled the rate, in people per hour, at which people enter an auditorium during the two hours ($0 \le t \le 2$) prior to the start of a rock concert. It was stated that the auditorium was empty at time $t = 0$, and part (a) asked for the number of people in the auditorium at time $t = 2$, which required computation of the definite integral $\int_0^2 R(t)\,dt$. In part (b) students needed to find the time t that maximizes $R(t)$. Part (c) defined the total wait time for all the people in the auditorium and stated that a function w that models the total wait time for all the people who entered the auditorium by time t has derivative $w'(t) = (2 - t)R(t)$. Students were asked to evaluate $w(2) - w(1)$ and should have recognized that this is computed by $\int_1^2 w'(t)\,dt$. Part (d) asked for the average amount of time that a concertgoer spent waiting for the concert to begin after entering the auditorium. Students needed to compute the total wait time, $\int_0^2 w'(t)\,dt$, for all people attending the concert and divide this by the number of people in the auditorium at the start of the concert as found in part (a).

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), 2 points in part (b), 2 points in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the first 2 points by correctly computing $R'(t)$ and determining the correct interior critical point. The student considers the sign change of R' at $t = 1.363$, providing an argument for a local maximum instead of a global maximum, and did not earn the third point. In part (c) the student's work is correct. In part (d) the student computes the average value of $w'(t)$ over the interval from 1 to 2 instead of the total wait time $w(2)$ divided by the total number of people.

Sample: 2C
Score: 4

The student earned 4 points: 2 points in part (a), 1 point in part (b), 1 point in part (c), and no points in part (d). In part (a) the student's work is correct. In part (b) the student earned the first point for correctly computing $R'(t)$. The student's value for the critical point is incorrect, so the response was not eligible for the third point. In this case, no justification for a global maximum is given. In part (c) the student earned the first point for providing the correct definite integral for $w(2) - w(1)$. The student does not compute the value of the integral. In part (d) the student does not provide a definite integral for the numerator.