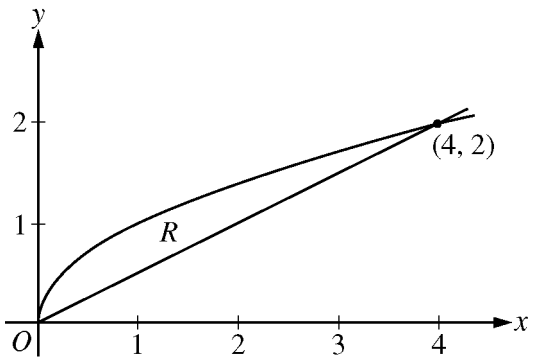
AP® CALCULUS AB
2009 SCORING GUIDELINES (Form B)

Question 4

Let R be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{2}$, as shown in the figure above.



(a) Find the area of R.

(b) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 2$.

(a) Area $= \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big|_{x=0}^{x=4} = \frac{4}{3}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(b) Volume $= \displaystyle\int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - x^{3/2} + \frac{x^2}{4}\right) dx$

$= \displaystyle\frac{x^2}{2} - \frac{2x^{5/2}}{5} + \frac{x^3}{12}\Big|_{x=0}^{x=4} = \frac{8}{15}$

3 : { 1 : integrand; 1 : antiderivative; 1 : answer }

(c) Volume $= \pi \displaystyle\int_0^4 \left(\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right) dx$

3 : { 1 : limits and constant; 2 : integrand }

NO CALCULATOR ALLOWED

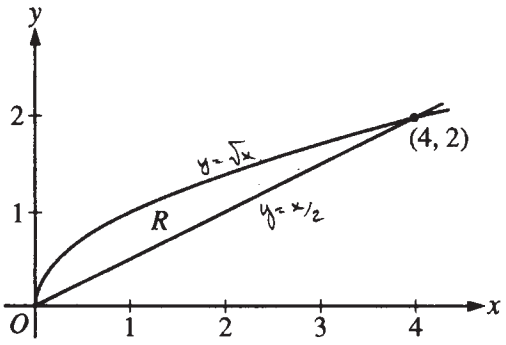
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

$$R = \int_0^4 (\sqrt{x} - x/2)\,dx \Rightarrow \int_0^4 x^{1/2} - x/2\,dx \Rightarrow \left[\frac{2x^{3/2}}{3} - x^2/4\right]_0^4$$

$$\Rightarrow \frac{2(4)^{3/2}}{3} - \frac{(4)^2}{4} \Rightarrow \frac{2\sqrt{64}}{3} - \frac{16}{4} \Rightarrow \frac{16}{3} - 4 \Rightarrow \frac{16-12}{3}$$

$$\Rightarrow \boxed{4/3}$$

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$$V = \int_0^4 (\sqrt{x} - x/2)^2 \, dx \Rightarrow \int_0^4 x - x^{3/2} + x^2/4 \, dx \Rightarrow \left[x^2/2 - 2x^{5/2}/5 + x^3/12 \right]_0^4$$

$$\Rightarrow \left[\frac{4^2}{2} - \frac{2(4)^{5/2}}{5} + \frac{(4)^3}{12} \right] \Rightarrow \frac{16}{2} - \frac{2\sqrt{(4)^5}}{5} + \frac{64}{12} \Rightarrow 8 - \frac{64}{5} + \frac{16}{3} \Rightarrow \boxed{\frac{8}{15}}$$

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Work for problem 4(c)

$y = \sqrt{x} \Rightarrow x = y^2$ $\quad y = \frac{x}{2} \Rightarrow x = 2y$.

$$V = \pi \int_0^4 (2 - x/2)^2 - (2 - \sqrt{x})^2 \, dx$$

GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

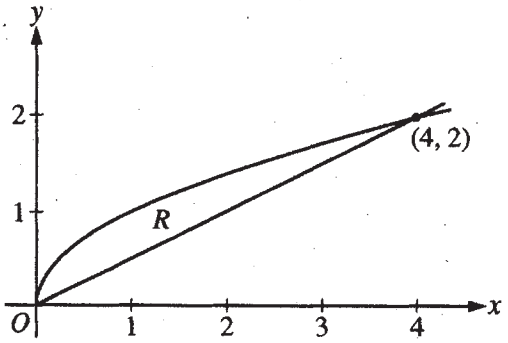
CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

No calculator is allowed for these problems.



Work for problem 4(a)

a) $A_R = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx = \int_0^4 \sqrt{x}\,dx - \int_0^4 \frac{x}{2}\,dx$

$\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 - \left[\frac{x^2}{4}\right]_0^4 = \left[\frac{2}{3}\sqrt{4^3} - 0\right] - \left[\frac{16}{4} - 0\right]$

$= \frac{2}{3}\sqrt{48} - 4 =$

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Continue problem 4 on page 11.

Work for problem 4(b)

$A_{square} = s^2 \qquad s = \sqrt{x} - \frac{x}{2}$

$$V = \int_0^4 s^2\,ds = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right)^2 dx = \int_0^4 \left(x - \frac{x^2}{4}\right) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^4 = \left[\left(8 - \frac{48}{12}\right) - 0\right] = 8 - 4 = 4\,u^3$$

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Work for problem 4(c)

axis of Rev. $\Rightarrow$ $y = 2$ // to x-axis dx.

Washer Method:

$R(x) = 2 - \frac{x}{2}$

$r(x) = 2 - \sqrt{x}$

$$V = \pi \int_0^4 \left[\left(2 - \frac{x}{2}\right)^2 - \left(2 - \sqrt{x}\right)^2\right] dx$$

NO CALCULATOR ALLOWED

CALCULUS AB

SECTION II, Part B

Time—45 minutes

Number of problems—3

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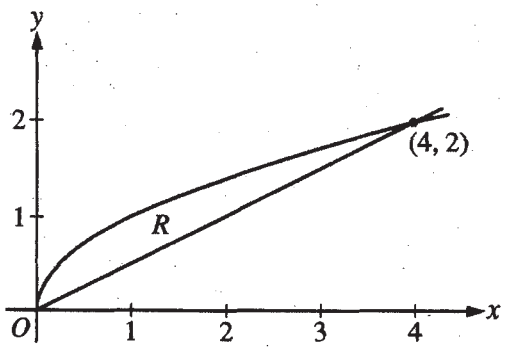

Work for problem 4(a)

$\sqrt{x} = 2$.

$Y_1 = \sqrt{x}$, $Y_2 = \frac{x}{2}$, $Y_1 = Y_2 \Rightarrow \sqrt{x} = \frac{x}{2} \Rightarrow x = 4$

$\Rightarrow$ ~~at $x = 2$~~ Area $R = \int_0^4 \sqrt{x} - \frac{x}{2}\,dx$

$x^{1/2}$.

$\Rightarrow$ Area $R = \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big]_0^4 dx$.

$\frac{2 \times 4 \times 2}{3} - \frac{16}{4} - 0 = \frac{16}{3} - \frac{16}{4} = \frac{16}{3} - 4 = \frac{16}{3} - \frac{12}{3}$.

$= 4/3$ units^2.

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Continue problem 4 on page 11.

NO CALCULATOR ALLOWED

Work for problem 4(b)

$V = s^3 \Rightarrow$ since square $\int_0^4 (\sqrt{x} - \frac{x}{2})^3$ ~~dx~~ $d/dx \sqrt{x} - \frac{x}{2} = (\frac{1}{2\sqrt{x}} - \frac{1}{2})^3$

$\Rightarrow \int_0^4 (\frac{1}{2\sqrt{x}} - \frac{1}{2})^3 .$

Work for problem 4(c)

$V = \pi \int_0^4 R^2 - r^2 \, dx = \pi \int_0^4 (\sqrt{x})^2 - (\frac{x}{2})^2 \, dx. = \pi \int_0^4 x - \frac{x^2}{4} \, dx.$

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AP® CALCULUS AB
2009 SCORING COMMENTARY (Form B)

Question 4

Sample: 4A
Score: 9

The student earned all 9 points.

Sample: 4B
Score: 6

The student earned 6 points: 2 points in part (a), 1 point in part (b), and 3 points in part (c). In part (a) the student earned the first 2 points, but the answer is incorrect. In part (b) the student earned the integrand point. The student has an algebra error that leads to an incorrect antiderivative and was not eligible for the answer point. In part (c) the student's work is correct.

Sample: 4C
Score: 4

The student earned 4 points: 3 points in part (a), no points in part (b), and 1 point in part (c). In part (a) the student's work is correct. In part (b) the student has an incorrect integrand. In part (c) the student earned the limits and constant point.