# AP® CALCULUS AB
# 2009 SCORING GUIDELINES (Form B)

## Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \le t \le 3$, where time $t$ is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time $t$ is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \le t \le 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\, dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

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|---|---|
| (a) $R(t) = 6 + \int_0^t \frac{1}{16}\left(3 + \sin\left(x^2\right)\right) dx$<br>$R(3) = 6.610$ or $6.611$ | $3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for } R(t) \\ 1 : R(3) \end{cases}$ |
| (b) $A(t) = \pi\left(R(t)\right)^2$<br>$A'(t) = 2\pi R(t) R'(t)$<br>$A'(3) = 8.858 \text{ cm}^2/\text{year}$ | $3 : \begin{cases} 1 : \text{expression for } A(t) \\ 1 : \text{expression for } A'(t) \\ 1 : \text{answer with units} \end{cases}$ |
| (c) $\int_0^3 A'(t)\, dt = A(3) - A(0) = 24.200$ or $24.201$<br><br>From time $t = 0$ to $t = 3$ years, the cross-sectional area grows by 24.201 square centimeters. | $3 : \begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{value of } \int_0^3 A'(t)\, dt \\ 1 : \text{meaning of } \int_0^3 A'(t)\, dt \end{cases}$ |

**CALCULUS AB**

**SECTION II, Part A**

**Time—45 minutes**

**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

Work for problem 1(a)

$R(t) = \int_0^t \frac{1}{16}(3+\sin(x^2))\,dx + 6$ for $0 \le t \le 3$.

Thus $R(3) = \int_0^3 \frac{1}{16}(3+\sin(x^2))\,dx + 6 = 0.611 + 6 = 6.611$

Work for problem 1(b)

$A'(t) = \frac{dA}{dt} = \frac{dA}{dR} \cdot \frac{dR}{dt} = 2\pi R(t) \cdot \frac{1}{16}(3+\sin(t^2)) = \frac{\pi R(t)}{8}(3+\sin(t^2))$

Hence at $t=3$ years, $A'(3) = \frac{\pi R(3)}{8}(3+\sin(3^2)) = 8.858$.

The unit of measure is centimeter$^2$/year.

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Work for problem 1(c)

$\int_0^3 A'(t)\,dt = A(t)\Big|_0^3 = A(3) - A(0) = \pi(R(3))^2 - \pi(R(0))^2$

$= 24.207$

The unit is $cm^2$, and this intergral means the growth of the area of the cross section from year 0 to year 3.

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**CALCULUS AB**

**SECTION II, Part A**

**Time—45 minutes**

**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

Work for problem 1(a)

$\int dR = \frac{1}{16} \int (3 + \sin t^2) dt$

$R = \frac{1}{16} \int (3 + \sin t^2) dt + 6$

$R(3) = \frac{1}{16} \int_0^3 (3 + \sin t^2) dt + 6 = 6.611 \text{ cm}$

Work for problem 1(b)

$A(t) = \pi r^2 = \pi (R(t))^2$

$\frac{dA}{dt} = 2\pi R(t) \cdot \frac{dR}{dt}$

$\left.\frac{dA}{dt}\right|_3 = 2\pi \cdot R(3) \cdot \left.\frac{dR}{dt}\right|_3 = 8.858 \text{ cm}^2/\text{yr}$

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Work for problem 1(c)

$\int_0^3 A'(t)dt = \int_0^3 \frac{dA}{dt} = A(3) - A(0)$

$A(3) = \pi(R(3))^2 = 137.298$

$A(0) = \pi(6)^2 = 36\pi$

$A(3) - A(0) = 24.201 \text{ cm}^2$

$\int_0^3 A'(t)dt = 24.201 \text{ cm}^2$ is the ~~a~~ cross-sectional area of the trunk at $t = 3$ yrs.

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 1C₁

**CALCULUS AB**

**SECTION II, Part A**

**Time—45 minutes**

**Number of problems—3**

**A graphing calculator is required for some problems or parts of problems.**

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Work for problem 1(a)

$\frac{dR}{dt} = \frac{1}{16}\left(3+\sin(t^2)\right) \Rightarrow dR = \left(\frac{1}{16}(3+\sin t^2)\right)dt$

$R(t) = \int_0^3 \left[\frac{1}{16}(3+\sin t^2)\right] dt \Rightarrow R(3) - R(0) = \int_0^3 \left[\frac{1}{16}(3+\sin t^2)\right] dt$

$R(3) - 6 = 0.611 \Rightarrow R(3) = 6.611$ centimeters

Work for problem 1(b)

$A = \pi r^2 \; ; \; r = R(t)$

$\frac{dA}{dr} = \frac{dA}{dt}\cdot\frac{dt}{dr} \Rightarrow \frac{dA}{dt} = \frac{dA}{dr}\cdot\frac{dr}{dt} \Rightarrow \frac{dA}{dt} = 2\pi r \cdot \left(\frac{1}{16}(3+\sin t^2)\right)$

when $t=3 \Rightarrow \frac{dA}{dt} = 2\pi(3)\left(\frac{1}{16}(3+\sin 3^2)\right) = 4.020 \ \frac{cm^2}{year}$

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Work for problem 1(c)

$$A'(t) = \frac{dA}{dt} = \frac{\pi r}{8}(3 + \sin t^2) \Rightarrow \int_0^3 A'(t)\,dt = \int_0^3 \frac{\pi r}{8}(3 + \sin t^2)\,dt$$

$$= 5.677 \text{ cm}^2$$

this integral shows us the difference between the cross-sectional area In the first 3 years, in other words it shows us the increase of the Cross-sectional Area in the first 3 years. in $Cm^2$.

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## Question 1

**Sample: 1A**
**Score: 9**

This student earned all 9 points. Note that in part (b) the student earned the $A(t)$ point implicitly. In part (c) the student earned the answer point in spite of using a rounded value for $R(3)$ from part (a).

**Sample: 1B**
**Score: 6**

The student earned 6 points: 1 point in part (a), 3 points in part (b), and 2 points in part (c). In part (a) the student only earned the point for $R(3)$ since an integral expression for $R(t)$ is not included. In part (b) the student's work is correct. In part (c) the student earned the first 2 points. The student did not earn the point for the meaning of the definite integral since the response mentions cross-sectional area at a particular time rather than growth in cross-sectional area over the three-year period.

**Sample: 1C**
**Score: 4**

The student earned 4 points: 1 point in part (a), 2 points in (b), and 1 point in (c). In part (a) the student only earned the point for $R(3)$ since an integral expression for $R(t)$ is not included. In part (b) the last equality on the student's second line earned the first 2 points. Although $A(t)$ is not explicitly stated, the student earned the $A(t)$ point. The student did not earn the answer point since 3 is used, instead of $R(3),$ in the calculation of $\frac{dA}{dt}$ at $t = 3.$ In part (c) the student earned the point for the meaning of the definite integral.