Question 6

Intent of Question

The primary goals of this investigative task were to assess a student's ability to (1) identify and conduct an appropriate inference based on the differences in the posttest and pretest scores; (2) identify and interpret appropriate information from statistical software; (3) make an inference based on separate regression analyses; and (4) recognize and explain the additional information provided from the different analyses.

Solution

Part (a):

Component 1: States a correct pair of hypotheses.

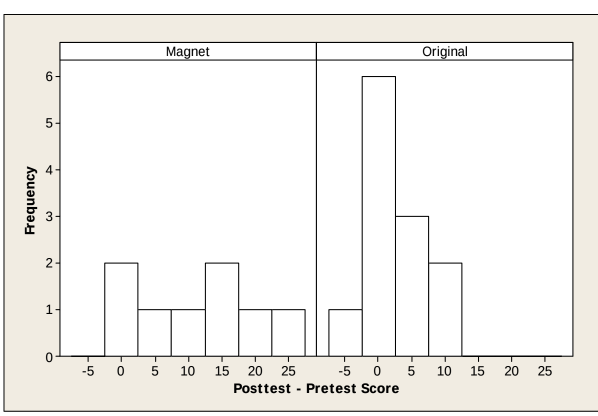
We want to test $H_0 : \mu_{DiffM} = \mu_{DiffO}$ versus $H_a : \mu_{DiffM} > \mu_{DiffO}$, where μ_{DiffM} is the mean difference (posttest − pretest) for all students at the magnet school and μ_{DiffO} is the mean difference (posttest − pretest) for all students who applied to attend the magnet school but were not selected and then attended the original school.

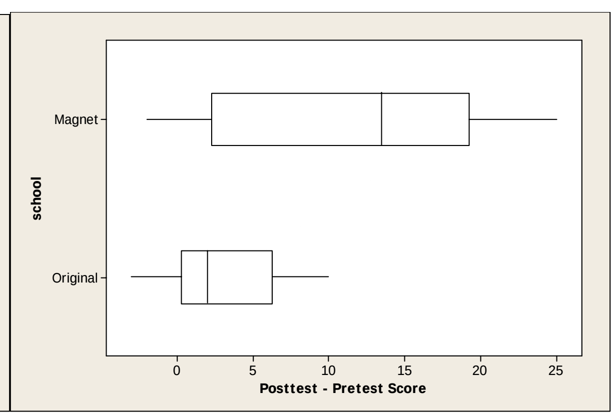
Component 2: Identifies a correct test (by name or formula) and checks the conditions.

A two-sample t-test for means, or $t = \dfrac{\overline{x}_M - \overline{x}_O}{\sqrt{\dfrac{s_M^2}{n_M} + \dfrac{s_O^2}{n_O}}}$

1. We need to assume randomness of the sampling used. It was stated in the stem that the students from the two different schools were randomly selected.

2. We need to check the assumption that the distributions of differences (posttest – pretest) for each of the two schools are normally distributed. Based on histograms and boxplots of these differences, there are no outliers or extreme skewness. Because these graphs reveal no obvious departures from normality, it appears reasonable to proceed with the t-test.





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Question 6 (continued)

Component 3: Performs correct mechanics, which include the value of the test statistic and p-value (or rejection region): $t = \dfrac{\bar{x}_M - \bar{x}_O}{\sqrt{\dfrac{s_M^2}{n_M} + \dfrac{s_O^2}{n_O}}} = \dfrac{11.75 - 3}{\sqrt{\dfrac{88.55}{8} + \dfrac{15.84}{12}}} \approx 2.487$
with a (one-sided) p-value ≈ 0.0177, df ≈ 8.69.

Component 4: Draws an appropriate conclusion in context and with linkage to the p-value (or rejection region):

Using $\alpha = 0.05$, we reject H_0 because $0.0177 < 0.05$. We conclude that the sample data provide convincing evidence that students who attend the magnet school have a higher mean difference in test scores than students who attend the original school.

Part (b):

Let y = posttest score and x = pretest score.

(i). The predicted regression equation for the magnet school is $\hat{y} = 73.27 + 0.1811x$. For students at the magnet school, a 1-point increase in the pretest score is associated with a predicted increase of 0.1181 points on the posttest (i.e., the slope is positive but close to zero).

(ii). The predicted regression equation for the original school is $\hat{y} = 9.24 + 0.9204x$. For students at the original school, a 1-point increase in the pretest score is associated with a predicted increase of 0.9204 points on the posttest (i.e., the slope is positive and close to 1).

Part (c):

(i). The test statistic is $t = 0.40$ with a ***p*-value of 0.706**. Because the p-value is greater than any reasonable significance level, say 0.05, we fail to reject H_0. We conclude that there is insufficient evidence to state that pretest score is a significant predictor of posttest score at the magnet school. The data do not support a conclusion that a correlation exists between pretest and posttest scores at the magnet school.

(ii). The test statistic is $t = 6.09$ with a ***p*-value of 0.000**. Because the p-value is less than any reasonable significance level, say 0.05, we reject H_0 and conclude that there is sufficient evidence to state that pretest score is a significant predictor of posttest score at the original school. The data support a conclusion that a correlation exists between pretest and posttest scores at the original school.

Part (d):

Unlike the two-sample analysis of differences in part (a), the regression analyses allow us to explore the relationship between pretest and posttest scores at each school. From the regression output and graph, we see that students with low pretest scores benefit more from attending magnet schools, as compared with students with low pretest scores at the original school. Also at the magnet school, students with low pretest scores benefit more than students with high pretest scores. In other words, students at the magnet school all score high on the posttest, regardless of how they scored on the pretest. But at the original school, only students who scored high on the pretest scored high on the posttest.

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Question 6 (continued)

Scoring

Parts (a), (b), (c), and (d) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if all four components are correct.

Partially correct (P) if two or three components are correct.

Incorrect (I) if at most one component is correct.

Part (b) is scored as follows:

Essentially correct (E) if all four components—both equations and both interpretations in (i) and (ii)—are correct.

Partially correct (P) if two or three components are correct.

Incorrect (I) if at most one component is correct.

Part (c) is scored as follows:

Essentially correct (E) if all four components—both *p*-values and both conclusions in (i) and (ii)—are correct.

Partially correct (P) if two or three components are correct.

Incorrect (I) if at most one component is correct.

Part (d) is scored as follows:

Essentially correct (E) if the response clearly explains how the regression analyses provide additional information in this context **by addressing the impact of the magnet school on students with low pretest scores.**

Partially correct (P) if the response clearly describes how the regression analyses provide additional information in context but does not explain the impact of the magnet school on students with low pretest scores.

Incorrect (I) if the response does not meet the criteria for an E or P.

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Question 6 (continued)

Each essentially correct (E) response counts as 1 point. Each partially correct (P) response counts as $\frac{1}{2}$ point.

4 **Complete Response**

3 **Substantial Response**

2 **Developing Response**

1 **Minimal Response**

If a response is between two scores (for example, $2\frac{1}{2}$ points) use a holistic approach to determine whether to score up or down, depending on the overall strength of the response and communication.

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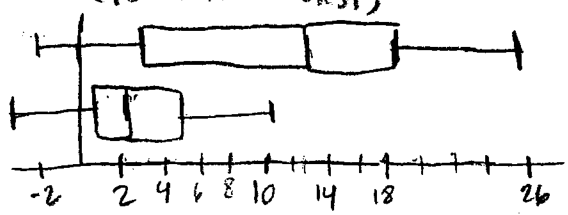
(a) Perform a test to determine whether students who attend the magnet school demonstrate a significantly higher mean difference in test scores (Posttest − Pretest) than students who applied to attend the magnet school but who were not selected and then attended their original school.

2 sample t-test.

Conditions

- Random selection of students from the magnet school and from the original school is given.
- The difference in test scores of one student should not affect the difference in scores of another student, so independence can be assumed.
- large enough sample to assume an approximately normal sampling distribution? sample size isn't greater than or equal to thirty, so we must examine the box plots of the sample

Differences in test scores
(Posttest - Pretest)



The box plots are symmetric enough.

$H_0: M_1 = M_2$

$H_A: M_1 > M_2$

M_1 = mean difference in test scores for students in the magnet school (post - pre)

M_2 = Mean difference in test scores for students that applied, but weren't accepted into the magnet program

$t = 2.487$

$df = 8.689$

$\alpha = .05$

p-value = .0177

Reject H_0 because the p-value is so low. We have enough evidence to conclude that the mean difference in test scores for student in the magnet program is greater than the mean difference in test scores for students that did not get in to the magnet program (posttest - pretest).

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(b) (i) State the equation of the regression line for the magnet school and interpret its slope in the context of the question.

$\hat{y} = .1811x + 73.27$

Slope = .1811
For every increase in one point on the pretest score, there will be an increase in .1811 points on the post test score, on average.

y-intercept = 73.27
A student that scores a zero on the pretest would be expected to score a 73.27 on the posttest.

(ii) State the equation of the regression line for the original school and interpret its slope in the context of the question.

$\hat{y} = .9204x + 9.24$

Slope = .9204
With every additional point scored on the pretest the student should expect to score .9204 points higher on the post test

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6A3

(c) To determine whether there is a significant correlation between pretest score and posttest score, a test of the following hypotheses will be performed.

H_0 : There is no correlation between pretest score and posttest score (true slope = 0)

versus

H_a : There is a correlation between pretest score and posttest score (true slope $\neq$ 0)

(i) Using the regression output, state the *p*-value and conclusion for this test at the magnet school. Assume the conditions for inference have been met.

p-value: .706

Fail to reject H_0. We do not have enough evidence to conclude that the true slope of the line is not zero. Therefore it may be zero and a linear model would not be appropriate

(ii) Using the regression output, state the *p*-value and conclusion for this test at the original school. Assume the conditions for inference have been met.

p-value = 0.0

Reject H_0. We have enough evidence to conclude that there is a linear correlation between pretest score and post test score

(d) What additional information do the regression analyses give you about student performance on the science test at the two schools beyond the comparison of mean differences in part (a)?

The regression analysis shows that a magnet school student that scores poorly on the pretest is likely to score much higher on the post test. For example, a student that scored a 65 on the pretest is expected to score an 85 on the post test after the magnet school program. A student at the original school doesn't experience as much progress. a student that scored a 65 on the pretest is only expected to score a 69 on the post test after the original school's curriculum

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(a) Perform a test to determine whether students who attend the magnet school demonstrate a significantly higher mean difference in test scores (Posttest − Pretest) than students who applied to attend the magnet school but who were not selected and then attended their original school.

$H_0: \mu_1 = \mu_2$ $\alpha = .05$

$H_a: \mu_1 > \mu_2$

$\bar{x}_1 = 11.750$ $\bar{x}_2 = 3.000$

$s_1 = 9.407$ $s_2 = 3.977$

$t = 2.4869$

$P(t_{8.6891} > 2.4869) =$ p-value $= .0177$

$.0177 < .05$

Reject H_0 at $\alpha = .05$ significance.

The data suggests that the students who attended the magnet school had a greater mean difference in test scores than those who did not.

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6B2

(b) (i) State the equation of the regression line for the magnet school and interpret its slope in the context of the question.

$\hat{y} = 73.27 + .1811x$

As pretest score increases by 1 point, post-test score is expected to increase by .1811 points.

(ii) State the equation of the regression line for the original school and interpret its slope in the context of the question.

$\hat{y} = 9.24 + .9204x$

As pretest score increases by 1 point, posttest score is expected to increase by .9204 points.

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(c) To determine whether there is a significant correlation between pretest score and posttest score, a test of the following hypotheses will be performed.

H_0 : There is no correlation between pretest score and posttest score (true slope = 0)
versus
H_a : There is a correlation between pretest score and posttest score (true slope $\neq$ 0)

(i) Using the regression output, state the *p*-value and conclusion for this test at the magnet school. Assume the conditions for inference have been met.

The p-value is .706, which is greater than .05, so the hypothess that there is no correlation between pretest and post-test scores cannot be rejected.

(ii) Using the regression output, state the *p*-value and conclusion for this test at the original school. Assume the conditions for inference have been met.

The p-value is .000, which is less than .05, so the hypothesis that there is no correlation between pretest and posttest scores is rejected.

(d) What additional information do the regression analyses give you about student performance on the science test at the two schools beyond the comparison of mean differences in part (a) ?

The analyses explain that the students at the magnet school responded in different ways—some saw their scores skyrocket, others found them remain relatively constant. The original school saw its students improve more consistent and predictably, even if the improvement was generally smaller.

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(a) Perform a test to determine whether students who attend the magnet school demonstrate a significantly higher mean difference in test scores (Posttest – Pretest) than students who applied to attend the magnet school but who were not selected and then attended their original school.

I. $H_0: \mu_{D(magnet)} = \mu_{D(original)}$

$H_a: \mu_{D(magnet)} > \mu_{D(original)}$

The mean difference in test scores of students of the magnet school is higher than the mean diff of students of the original school

The mean difference in test scores of students of the magnet school is greater than the mean diff of students in the original school.

II. (i) $n = 20$ is sufficiently large to be assumed normal
(ii) 2-sample T Test
(iii) simple random samples taken independently of eachother

(assumptions)

III. $t = 2.487$
$p = 0.0177$
$= 1.77\%$

IV. If the significance level $\alpha = 5\%$ is used, since $p = 1.77\% < \alpha = 5\%$, the null hypothesis can be rejected.

Therefore, there is statistical evidence that suggests the mean difference in test scores of students who attend the magnet school is higher than the mean difference in test scores of students who attended their original school.

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(b) (i) State the equation of the regression line for the magnet school and interpret its slope in the context of the question. 6C2

$y = 73.27 + 0.1811x$

slope $b = 0.1811$.

For every increase in X (point in the pretest scores), the posttest score increases by the slope: 0.1811 points.

(ii) State the equation of the regression line for the original school and interpret its slope in the context of the question.

$y = 9.24 + 0.9204x$

slope $b = 0.9204$.

For every increase in X (point in the pretest score), the posttest score increases by the slope: 0.9204 points.

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(c) To determine whether there is a significant correlation between pretest score and posttest score, a test of the following hypotheses will be performed.

H_0 : There is no correlation between pretest score and posttest score (true slope = 0)
versus
H_a : There is a correlation between pretest score and posttest score (true slope $\neq$ 0)

(i) Using the regression output, state the *p*-value and conclusion for this test at the magnet school. Assume the conditions for inference have been met.

$H_0: \beta = 0$ $p = 0.7063$

$H_a: \beta \neq 0$ Since $p > \alpha = 5\%$, the null hypo cannot be rejected.

Therefore, there is statistical evidence that suggests there is no correlation between pretest scores and posttest scores at the magnet school

(ii) Using the regression output, state the *p*-value and conclusion for this test at the original school. Assume the conditions for inference have been met.

$p = 1.175 \times 10^{-4}$

Since $p < \alpha = 5\%$, the null hypothesis can be rejected.

Therefore, there is statistical evidence that suggests there is a correlation between pretest scores and posttest scores at the original school.

(d) What additional information do the regression analyses give you about student performance on the science test at the two schools beyond the comparison of mean differences in part (a)?

The regression analyses suggests that although the mean differences hypothesis test concluded the students of the model school scored better, the students at the original school actually improved much more than those at the model school.

The slopes show that while the magnet school students' posttest score increased by just 0.18 points for every point increase on the pretest, the original school students' posttest score improved by 0.92 points.

Also, the $\beta = 0$ test suggests a very high correlation between the pre & posttest scores for the original school students also portray the high improvement

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Question 6

Overview

The primary goals of this investigative task were to assess a student's ability to (1) identify and conduct an appropriate inference based on the differences in the posttest and pretest scores; (2) identify and interpret appropriate information from statistical software; (3) make an inference based on separate regression analyses; and (4) recognize and explain the additional information provided from the different analyses.

Sample: 6A
Score: 4

This response is correct, complete, and well expressed in all parts. The hypotheses in part (a) are stated correctly, with the parameter symbols defined clearly as the means of the differences. The two-sample *t*-test is identified by name at the top of the page. The random sampling condition is mentioned, and the normality condition is checked by examining boxplots. The response indicates that the sample sizes are small enough to require normality of the differences in order to apply the two-sample *t*-test. The mechanics of calculating the test statistic and *p*-value are correct; notice that the formula for calculating the test statistic is not necessary. The conclusion is very well expressed in context, being carefully worded in terms of "the mean difference in test scores."

The equations in (b) are reported correctly, using good $\hat{y}$ notation to denote a predicted value. The variables are not clearly defined, but the response makes clear which variable (pretest score) is represented by x and which (posttest score) is represented by y. The interpretations of slope are both good. The interpretation in (i) uses the phrase "on average" to convey randomness/variability, while the interpretation in (b) uses the phrase "expect to."

The *p*-values in part (c) are reported correctly, and the conclusions are presented clearly. The response is not required to show linkage between the *p*-value and conclusion, because the question simply asks students to "state the *p*-value and conclusion" rather than to conduct a full hypothesis test.

The response to part (d) indicates what the regression analyses reveal about student performance on the science test at these two schools. The first sentence captures the essential point by observing that "a magnet school student that scores poorly on the pretest is likely to score much higher on the post test." The last three sentences of the response to this part give a specific example, pointing out that a student with a pretest score of 65 is expected to score an 85 on the posttest in the magnet school and only a 69 in the original school. The use of the words "likely" and "expected to" in these sentences indicates a recognition and understanding of regression lines as models.

This answer was judged complete in all its parts and especially impressive for its clear communication and concise description of the fundamental point in part (d). It merited 4 points.

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Question 6 (continued)

Sample: 6B
Score: 3

The response in part (a) contains vague hypotheses. The two groups should be defined as the magnet school and original school, respectively, and ideally the "μ_1" and "μ_2" symbols would be defined. The response makes no effort to identify the appropriate test (by name or formula), and no conditions are stated or checked. The test statistic and *p*-value are calculated correctly, and the conclusion is correct in context with linkage to the *p*-value. Part (a) was scored as partially correct.

The response for part (b) is quite good. The equation of the regression line is provided, carefully expressed in terms of $\hat{y}$ rather than simply y. The variables y and x should be defined, but the slope interpretation demonstrates an understanding of what each symbol represents. The phrase "expected to" is a good way to indicate randomness/variability in the slope interpretation. Part (b) was scored as essentially correct.

The response to part (c) correctly identifies the two *p*-values and states the appropriate conclusion in context. The *p*-values are compared to a significance level of $\alpha = 0.05$. Part (c) was scored as essentially correct.

The response to part (d) indicates that the magnet and original schools have different impacts on students, depending on their pretest scores. The student describes this in dramatic terms for the magnet school, writing that "some saw their scores skyrocket, others found them remain relatively constant." What is lacking in this response is an indication that the students with *low* pretest scores are the ones whose scores improve dramatically. Owing to this omission, part (d) was scored as partially correct.

This response contained two parts that were deemed essentially correct and two parts that were assessed as partially correct. Overall, the answer was considered substantial but not complete; it earned 3 points.

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Question 6 (continued)

Sample: 6C
Score: 2

The hypotheses in part (a) are presented correctly in symbols. The accompanying sentence is correct for the alternative hypothesis but not for the null hypothesis. The two-sample *t*-test is identified correctly, and the condition of “simple random samples” is mentioned. The normality condition is mentioned but not checked, so the component for identifying the test and checking conditions was not considered to be met. The mechanics are correct, as is the conclusion. Part (a) was scored as partially correct.

The equations reported in part (b) are correct, but the variable names are not provided. The slope interpretations make clear which variable is the pretest score and which is the posttest score. However, the interpretations make no mention of randomness/variability, making the relationships sound deterministic. Part (b) was scored as partially correct.

The *p*-values are reported correctly in part (c). In fact, it seems that the *p*-value was determined by using a calculator, because the *p*-values include more significant figures than the output provided. The conclusions are correct and in context. The hypotheses are provided, as is linkage between the *p*-value and conclusion, although neither of those is required here. The conclusion in (i) would be stronger if it were not essentially accepting H_0 by saying that “there is no correlation between pretest scores and posttest scores at the magnet school.” Despite this concern, part (c) was scored as essentially correct.

The response in part (d) contradicts the correct conclusion from part (a) by stating that “students at the original school actually improved much more than those at the model school.” The student appears to be confusing the steepness of the regression lines (for predicting posttest from pretest) with the amount of improvement (posttest – pretest). This response was scored as incorrect.

With one essentially correct part and two partially correct parts, this answer was considered a developing response and received 2 points.

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