AP® STATISTICS
2008 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals for this question were to assess a student's ability to (1) recognize and calculate the mean as the expected value of a probability distribution; (2) demonstrate how to use two distributions to form all possible ways a specific difference may occur; (3) calculate a probability for this specific difference occurring; and (4) calculate a probability from the probability distribution of all possible differences.

Solution

Part (a):

The expected scores are as follows:

Josephine

$\mu_J = 16(0.1) + 17(0.3) + 18(0.4) + 19(0.2) = 17.7$

Crystal

$\mu_C = 17(0.45) + 18(0.4) + 19(0.15) = 17.7$

Part (b):

J	C	Probability
16	17	(0.1)(0.45) = 0.045
17	18	(0.3)(0.40) = 0.12
18	19	(0.4)(0.15) = 0.06

Part (c):

The probability is

0.045 + 0.12 + 0.06 = 0.225

Part (d):

P(difference = –1) = 0.225 (from part c)
P(difference = –2) = 1 – 0.015 – 0.225 – 0.325 – 0.260 – 0.90 = 0.085

Distribution of Josephine – Crystal

Differences	–3	–2	–1	0	1	2
Probability	0.015	**0.085**	**0.225**	0.325	0.260	0.090

The probability that Crystal's score is higher than Josephine's score is

P(difference < 0) = 0.015 + 0.085 + 0.225 = 0.325

Scoring

This problem is scored in three sections. Section 1 consists of part (a). Section 2 consists of parts (b) and (c). Section 3 consists of part (d). Each section is scored as essentially correct (E), partially correct (P), or incorrect (I).

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Question 3 (continued)

Section 1 [part (a)] is scored as follows:

Essentially correct (E) if correct expected scores (means) are calculated for both Josephine and Crystal with appropriate calculations or formulas shown for at least one of the players.

Partially correct (P) if the student makes one of the following errors:
- Rounds both expected values to integers (e.g., approximately 18 or 17–18)
- Calculates only one player's score correctly with appropriate calculations or formula
- Uses nonuniversal calculator syntax with linkage to the values in the table to describe how the correct expected values for both players are calculated
- Shows correct work for the expected values but gives answers of 17.5 and 18 (the unweighted averages)
- Gives correct expected values but does not show the multiplications or does not show the additions

Incorrect (I) if two or more of the errors above are made *OR* if no justification is given for correct answers *OR* if both expected scores are calculated using an incorrect method *OR* if the expected values are not calculated.

Note: If the student shows correct work but has at most one minor arithmetic error and/or copies at most one probability incorrectly from the table, the student should not be penalized for these types of errors.

Section 2 [parts (b) and (c)] is scored as follows:

Essentially correct (E) if all five of the components below are correctly completed by the student:
- Lists all the score combinations that result in a difference of –1 in part (b)
- Calculates the probabilities correctly in part (b)
- Shows appropriate work or formula in part (b)
- Calculates the correct probability for the difference of –1 in part (c)
- Shows appropriate work or formula in part (c)

Partially correct (P) if three or four of the previous components are correct.

Incorrect (I) if at most two of the previous components are correct.

Notes:
- If a student gets incorrect answers for the three combinations that result in a difference of –1 but uses them correctly in part (c), the student can still get credit for the last two components if the resulting probability is between 0 and 1.
- If the student shows correct work but has at most one minor arithmetic error and/or copies at most one probability incorrectly from the table, the student should not be penalized for these types of errors.

Section 3 [part (d)] is scored as follows:

Essentially correct (E) if both of the components below are successfully done by the student:
- Completes the table correctly
- Calculates the correct probability that Crystal's score is higher than Josephine's score *AND* shows appropriate work or formula

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Question 3 (continued)

Partially correct (P) if only one of the components is correct.

Incorrect (I) if both components are incorrect.

Notes:

- It is possible to calculate P(difference = –2) = 0.085 by listing the two combinations that result in a difference of –2.

J	C	Probability
16	18	(0.1)(0.4) = 0.04
17	19	(0.3)(0.15) = 0.045

- If a student has an incorrect answer in part (c) but uses it correctly in part (d), then the P(difference = –2) must be 0.085 *OR* the probabilities in the table must add up to 1 to get credit for the first component.
- If any of the values in the table are less than 0 or greater than 1, then no credit will be given for the first component.
- If the student shows correct work but has at most one minor arithmetic error and/or copies at most one probability incorrectly from the table, the student should not be penalized for these types of errors.

4 **Complete Response**

All three sections essentially correct

3 **Substantial Response**

Two sections essentially correct and one section partially correct

2 **Developing Response**

Two sections essentially correct and no sections partially correct

OR

One section essentially correct and one or two sections partially correct

OR

Three sections partially correct

1 **Minimal Response**

One section essentially correct and no parts partially correct

OR

No sections essentially correct and two sections partially correct

3A1

3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

(a) Calculate the expected score for each player.

Josephine: $16(.1) + 17(.3) + 18(.4) + 19(.2) = \boxed{17.7}$

Crystal: $17(.45) + 18(.4) + 19(.15) = \boxed{17.7}$

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is -1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of -1, and calculate the probability for each combination.

Josephine 16 Crystal 17 $.1 \cdot .45 = \boxed{.045}$

Josephine 17 Crystal 18 $.3 \cdot .4 = \boxed{.12}$

Josephine 18 Crystal 19 $.4 \cdot .15 = \boxed{.06}$

GO ON TO THE NEXT PAGE

3A2

(c) Find the probability that the difference (Josephine minus Crystal) in their scores is -1.

$.045 + .12 + .06 = .225$

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	−3	−2	−1	0	1	2
Probability	0.015	.085	.225	0.325	0.260	0.090

$1 - .015 - .225 - .325 - .260 - .090 = .085$

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

$.225 + .085 + .015 = .325$

$.325 = P(\text{Crystal's score} > \text{Josephine's score})$

3B1

3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

19-18 = (.20)(.40)
18-17 = (.40)(.45

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

(a) Calculate the expected score for each player.

Josephines expected score: 17.7
(16 • .10) + (17 • .30) + (18 • .40) + (19 • .20) = 17.7

Crystal expected score is: 17.7
(17 • .45) + (18 • .40) + (19 • .15) = 17.7

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is −1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of −1, and calculate the probability for each combination.

17 - 18 = -1 (.30 * .40) = .12
18 - 19 = -1 (.40 * .15) = .06
16 - 17 = -1 (.10 * .45) = .045

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(c) Find the probability that the difference (Josephine minus Crystal) in their scores is −1.

(.30 * .40) = .12
(.40 * .15) = .06
(.10 * .45) = .045

.12 + .06 + .045 =

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	−3	−2	−1	0	1	2
Probability	0.015	.085	.225	0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

The probability that Crystal's score will be higher than Josephine's score is .35.

Prob of -2 =

.015
+ .225
+ .325
+ .260
+ .090
.915

1 − .915 = .085 (prob of -2)

3. A local arcade is hosting a tournament in which contestants play an arcade game with possible scores ranging from 0 to 20. The arcade has set up multiple game tables so that all contestants can play the game at the same time; thus contestant scores are independent. Each contestant's score will be recorded as he or she finishes, and the contestant with the highest score is the winner.

After practicing the game many times, Josephine, one of the contestants, has established the probability distribution of her scores, shown in the table below.

Josephine's Distribution				
Score	16	17	18	19
Probability	0.10	0.30	0.40	0.20

expected score of Josephine = $\bar{x}_J$

Crystal, another contestant, has also practiced many times. The probability distribution for her scores is shown in the table below.

Crystal's Distribution			
Score	17	18	19
Probability	0.45	0.40	0.15

expected score of Crystal = $\bar{x}_C$

(a) Calculate the expected score for each player.

$\bar{x}_J = 17.7$ - 1 Varstats L1, L2

$\bar{x}_C = 17.7$ - 1 Varstats L1, L2

(b) Suppose that Josephine scores 16 and Crystal scores 17. The difference (Josephine minus Crystal) of their scores is −1. List all combinations of possible scores for Josephine and Crystal that will produce a difference (Josephine minus Crystal) of −1, and calculate the probability for each combination.

Possibilities: 16-17, 17-18, 18-19

P(16-17) = (.1)(.45) = .045

P(17-18) = (.3)(.4) = .12

P(18-19) = (.4)(.15) = .06

GO ON TO THE NEXT PAGE.

(c) Find the probability that the difference (Josephine minus Crystal) in their scores is −1.

$P(-1) = P(16-17) + P(17-18) + P(18-19)$

$= .045 + .12 + .06$

$= \boxed{.225}$

(d) The table below lists all the possible differences in the scores between Josephine and Crystal and some associated probabilities.

Distribution (Josephine minus Crystal)						
Difference	−3	−2	−1	0	1	2
Probability	0.015	.085	.225	0.325	0.260	0.090

Complete the table and calculate the probability that Crystal's score will be higher than Josephine's score.

$P(1 \text{ or } 2) = .26 + .09 - (.26)(.09)$

$= \boxed{.3266}$

AP® STATISTICS
2008 SCORING COMMENTARY

Question 3

Overview

The primary goals for this question were to assess a student's ability to (1) recognize and calculate the mean as the expected value of a probability distribution; (2) demonstrate how to use two distributions to form all possible ways a specific difference may occur; (3) calculate a probability for this specific difference occurring; and (4) calculate a probability from the probability distribution of all possible differences.

Sample: 3A
Score: 4

In part (a) the student calculates both expected values correctly and shows the appropriate work, so section 1 was scored as essentially correct. In part (b) the student lists the three score combinations that would result in a difference of –1. The student also shows the multiplications and arrives at correct probabilities. In part (c) the student shows the addition of the three probabilities from part (b) and gives the correct probability for a difference of –1. Section 2, which consists of parts (b) and (c), was thus scored as essentially correct. In part (d), which constitutes section 3, the student completes the table correctly, showing work for the calculation of ".085." The student also shows the addition of the probabilities for the three outcomes that result in a higher score for Crystal and gets the correct sum. Section 3 was scored as essentially correct. The answer, inclusive of all three parts, was deemed a complete response and earned 4 points.

Sample: 3B
Score: 3

In part (a) the student calculates both expected values correctly and shows the appropriate work; thus section 1 was scored as essentially correct. In part (b) the student lists all three combinations that would yield a difference of –1. The student also shows the multiplications and gets correct probabilities. In part (c) the student repeats the calculations from part (b) and shows the addition of these probabilities but does not write the sum. However, the correct sum does appear in the correct place in the table in part (d), so full credit was given for part (c). Section 2, which is made up of parts (b) and (c), was thereby scored as essentially correct. In part (d) the student correctly completes the table and shows the work but incorrectly finds the probability that Josephine has a higher score (using the outcomes 1 and 2) instead of the probability that Crystal has a higher score (using the outcomes –3, –2, and –1). Also, no work is shown for this calculation. Section 3, which comprises part (d), was scored as partially correct. The overall answer was judged a substantial response and was awarded 3 points.

Sample: 3C
Score: 2

In part (a) the student calculates the two expected values correctly but offers no work or formula to support the calculations. The student clearly used a calculator to compute the expected values, but there is no linkage to the values in the table, and the expression "1 var stats L1, L2" is not universally accepted statistical notation. Section 1 was therefore scored as partially correct. In part (b) the student lists all three combinations that would produce a difference of –1 and also shows the multiplications and arrives at the correct probabilities. In part (c) the student shows the addition of the three probabilities from part (b) and writes the correct sum. Section 2, which consists of parts (b) and (c), was scored as essentially correct. In part (d), which makes up section 3, the student completes the table correctly. However, the student makes two mistakes when attempting to find the probability that Crystal has a higher score than Josephine. The first is using the wrong differences from the table. The student should use the differences –3, –2, and –1 instead of 1 and 2. The second mistake is using the wrong formula. In this problem, the outcomes in the table are mutually exclusive, so nothing should be subtracted from the sum. Section 3 was scored as partially correct. The whole answer was considered a developing response and received 2 points.