AP® STATISTICS
2008 SCORING GUIDELINES (Form B)

Question 2

Intent of Question

The primary goals of this question were to assess a student's ability to (1) recognize an unbiased estimator and explain why the estimator is unbiased and (2) compare two estimators with respect to center and variability.

Solution

Part (a):

Statistics A, C, and D appear to be unbiased. This is indicated by the fact that the mean of the estimated sampling distribution for each of these statistics is about 75, the value of the true population parameter.

Part (b):

Statistic A would be a better choice because it appears to be unbiased. Although the variability of the two estimated sampling distributions is similar, statistic A would produce estimates that tend to be closer to the true population parameter value of 75 than would statistic B.

Part (c):

Statistic C would be a better choice because it has smaller variability. Although both statistic C and statistic D appear to be unbiased, statistic C would produce estimates that tend to be closer to the true population parameter value of 75 than would statistic D.

Scoring

Parts (a), (b), and (c) are each scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response contains the following two components.

> Component 1: Identifies statistics A, C, and D as the unbiased estimators.
> Component 2: Clearly demonstrates an understanding of the meaning of the term *unbiased*. That is, states that the mean (or center) of each distribution is about 75. No other characteristic (e.g., shape, spread) should be mentioned in the response unless it clearly is discounted as a criterion for being unbiased.

Partially correct (P) if the response contains just one of these components. That is, the response either identifies statistics A, C, and D as the unbiased estimators but gives a weak or no explanation or includes some discussion of another characteristic (e.g., shape, spread) as of some importance in judging bias *OR* demonstrates clear understanding of the meaning of the term *unbiased* but identifies only one or two of statistics A, C, and D as unbiased estimators.

Incorrect (I) otherwise.

Part (b) is scored as follows:

Essentially correct (E) if the response gives a clear explanation that statistic A is the better choice because it is centered at 75. (It is not necessary that the term *unbiased* be used as long as the response clearly addresses the center. It is not necessary to mention that statistics A and B both have about the same variability.)

Note: Students who were penalized in part (a) for mentioning other characteristics should not be repenalized in part (b).

Partially correct (P) if the response shows an understanding that centering is an important issue in determining which statistic is better but either does not apply the concept correctly *OR* communicates the explanation poorly.

Incorrect (I) otherwise.

Part (c) is scored as follows:

Essentially correct (E) if the response gives a clear explanation that statistic C is the better choice based on the fact that it has less variability. (It is not necessary to mention that statistics A and B are both centered at the parameter.)

Partially correct (P) if the response shows an understanding that variability is an important issue in determining which statistic is better but either does not apply the concept correctly *OR* communicates the explanation poorly.

Incorrect (I) otherwise.

4   Complete Response

> All three parts essentially correct

3   Substantial Response

OR
> Two parts essentially correct and the other part partially correct
>
> Part (a) and one other part essentially correct and the remaining part incorrect

2   Developing Response

OR
> Part (a) essentially correct and one of the other parts partially correct
>
OR
> All three parts partially correct
>
> Part (a) partially correct and one of parts (b) and (c) essentially correct

1   **Minimal Response**

> One part essentially correct and the other parts incorrect

OR

> Two parts partially correct and the other part incorrect

OR

> Part (a) incorrect, one of the other parts essentially correct, and one part partially correct

(a) Which of the statistics appear to be unbiased estimators of the population parameter?

How can you tell?

For an <u>unbiased estimator</u>, there is no <u>systematic tendency</u> for it to under-estimate or over-estimate the parameter. This means that the <u>mean of its sampling distribution</u>, μ_{stat}, must equal the parameter itself. For statistic $\boxed{A, C, \text{and } D}$ it appears that $\mu_{stat} = $ parameter $= 75$, so they are the ones that are unbiased estimators.

(b) Which of statistics A or B would be a better estimator of the population parameter?

Explain your choice.

μ_{stat} for B is somewhere around 85, and is clearly greater than 75.

μ_{stat} for A is very close to 75 (if not exactly).

So $\boxed{A}$ is better. (A good estimator is both unbiased and with low variability. Both A & B have low variability, but only A is unbiased)

(c) Which of statistics C or D would be a better estimator of the population parameter?

Explain your choice.

A good estimator is both <u>UNBIASED</u> and has <u>low VARIABILITY</u>.

Both C and D are unbiased, but D has much greater variability than C does.

So, $\boxed{C}$ is better.

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(a) Which of the statistics appear to be unbiased estimators of the population parameter?

Statistics A, C, +D

How can you tell?

The mean of the distributions appear to lie around 75.

(b) Which of statistics A or B would be a better estimator of the population parameter?

A

Explain your choice.

Statistic A is less biased than statistic B. B is too highly biased.

(c) Which of statistics C or D would be a better estimator of the population parameter?

C.

Explain your choice.

Though the shape of the graph is hanging in the middle, the mean of graph C would appear to fall around 75. Statistic D has a very high variance.

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(a) Which of the statistics appear to be unbiased estimators of the population parameter?

How can you tell?

Statistic A, which appears to be a normal distribution with mean around 75, appears to be an unbiased estimator.

(b) Which of statistics A or B would be a better estimator of the population parameter?

Explain your choice.

Statistic B is not centered at the population parameter, Statistic A is. Statistic A would be a better estimator of the population parameter

(c) Which of statistics C or D would be a better estimator of the population parameter?

Explain your choice.

Both statistics C and D are appropriately centered, but Statistic D has a very large range, making Statistic C the better estimator of the population parameter.

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AP® STATISTICS
2008 SCORING COMMENTARY (Form B)

Question 2

Sample: 2A
Score: 4

In part (a) this strong response correctly defines an unbiased estimator as one in which "the mean of its sampling distribution must equal the [population] parameter" and then explains why statistics A, C, and D apparently are unbiased estimators. Part (a) was scored as essentially correct. For parts (b) and (c), the student defines a good estimator as being "both unbiased and with low variability." Thus, although both statistics have about the same variability, statistic A is the better choice in part (b) because the mean of its estimated sampling distribution is closer to 75. Part (b) was scored as essentially correct. Although the estimated sampling distribution of both statistics have a mean of about 75, the student indicates that statistic C is the better choice in part (c) because its distribution has less variability. Part (c) was scored as essentially correct. The whole answer, with all three parts correct, was judged a complete response.

Sample: 2B
Score: 3

In part (a) this response gives the correct reason for selecting statistics A, C, and D as being the apparently unbiased estimators. Part (a) was scored as essentially correct. The choice of statistic A in part (b) is right, with an acceptable explanation, so part (b) was scored as essentially correct. The choice of statistic C in part (c) is also correct, but the reason for this choice is not clear. The two distributions are described separately, but no comparison is made between the two. Part (c) was scored as partially correct. The overall answer was deemed a substantial response.

Sample: 2C
Score: 2

This clearly written response does not reflect an understanding of the definition of an unbiased estimator. It includes the correct criterion in part (a) but also incorrectly implies in the explanation and by selecting only statistic A that normality is a second necessary requirement. Parts (b) and (c) are well done. Because part (a) was worth as much as parts (b) and (c) combined, this answer was considered a developing response.