Boolean Algebra Applications

Boolean algebra can be applied to any system in which each variable has two states. This chapter closes with sample problems solved by Boolean algebra.

EXAMPLE 1

Coffee, Tea, or Milk?

Snerdley's Automated Cafeteria orders a machine to dispense coffee, tea, and milk. Design the machine so that it has a button (input line) for each choice and so that a customer can have *at most one* of the three choices. Diagram the circuit to insure that the "at most one" condition is met.

Solution

Step 1 Specify the input and output variables and the two states of each.

Input c = coffee button (1 = pushed, 0 = not pushed)
t = tea button (1 = pushed, 0 = not pushed)
m = milk button (1 = pushed, 0 = not pushed)

Output x = choice verifier (1 = acceptable input–deliver the choice selected; 0 = unacceptable input–light an error light)

Step 2 Construct the truth table giving the output desired for each input.

x is 1 when exactly *one* of c, t, and m is 1.

c	t	m	x
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	1
0	0	0	0

Step 3 Write a Boolean expression with a term for each 1 output row of the table.

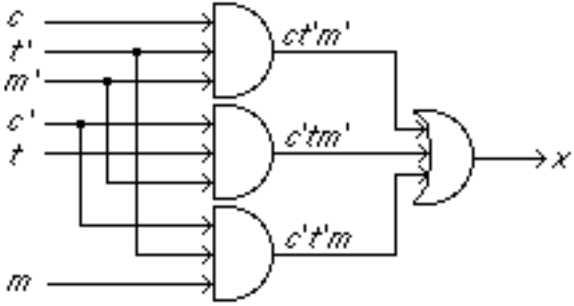
$$ct'm' + c'tm' + c't'm$$

Step 4 Try to simplify the formula.

	t		t'	
c				1
c'	1		1	
	m'	m		m'

The map shows that the expression cannot be simplified.

Step 5 Draw the circuit (next page).



EXAMPLE 2

U.S. Rocket Launcher

The nation of Upper Slobovia has gained a missile defense capability governed by its Security Council. The Council consists of four members: the U.S. (Upper Slobovian) President and three Counselors (the Chiefs of Staff of the Army and Air Force plus the President's Uncle Homer). The missile system is to be activated by a device obeying these rules: each member of the Security Council has a button to push; the missiles fire only if the President and at least one Counselor push their buttons. Design the rocket firing circuitry.

Solution

Step 1 Specify the input and output variables and the two states of each.

Input p = President's button (1 = pushed, 0 = not pushed)
x, y, z = Counselors' buttons (1 = pushed, 0 = not pushed)

Output f = fire missiles command (1 = fire, 0 = don't fire)

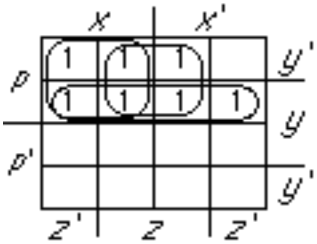
Step 2 Construct the truth table listing all possibilities.

	p	x	y	z	f	
The President and at least one Counselor agree: fire the missiles!	1	1	1	1	1	→ $pxyz$
	1	1	1	0	1	→ $pxyz'$
	1	1	0	1	1	→ $pxy'z$
	1	1	0	0	1	→ $pxy'z'$
	1	0	1	1	1	→ $px'yz$
	1	0	1	0	1	→ $px'yz'$
	1	0	0	1	1	→ $px'y'z$
Only the President pushed his button: do not fire.	1	0	0	0	0	
The President did not push his button: do not fire.	0	1	1	1	0	
	0	1	1	0	0	
	0	1	0	1	0	
	0	1	0	0	0	
	0	0	1	1	0	
	0	0	1	0	0	
	0	0	0	1	0	
	0	0	0	0	0	

Step 3 Write a Boolean expression.

$$pxyz + pxyz' + pxy'z + pxy'z' + px'yz + px'yz' + px'y'z$$

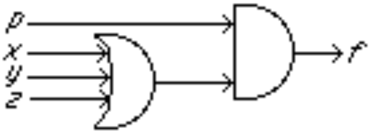
Step 4 Simplify the formula.



The map shows three groups of four 1’s each, giving this expression.

$$px + py + pz \text{ or } p\,(x + y + z)$$

Step 5 Draw the circuit.



EXAMPLE 3

Two-Floor Elevator

Numerous functions must be performed by the circuitry of an elevator (open/close door, move up/down, light up/down indicator, and so on). This example focuses on one aspect of a two-floor elevator: deciding when to move to the other floor.

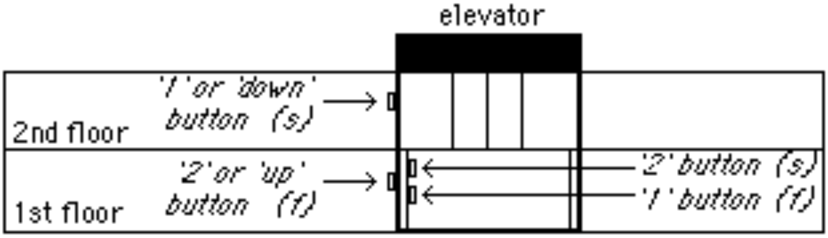
Solution

Step 1 Three inputs are needed.

f = first floor button (1 = pushed, 0 = not pushed)

s = second floor button (1 = pushed, 0 = not pushed)

p = present floor indicator (1 = 1st floor, 0 = 2nd floor)



For *f*, as the diagram above illustrates, there are two buttons: one outside the elevator on the first floor and one inside the elevator. Assume that these two buttons are connected in parallel to one line into the system. Similarly for *s*, assume the “2” buttons inside the elevator and outside the elevator on the second floor are connected.

Output *m* = “move” function (1 = move or change floor; 0 = stay)

Step 2 The truth table is on the next page.

The first two input combinations mean that both buttons have been pushed. So stay on the same floor and load passengers there first (that is, *m* = 0).

f	s	p	m
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

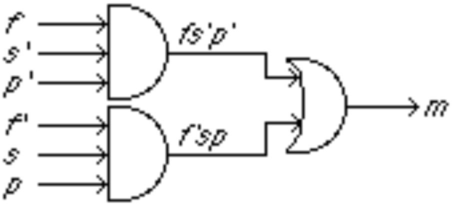
→ Elevator is on the 2nd floor and the 1st floor button is pushed; so move.

Elevator is on the first floor and the 2nd floor button is pushed–move. ←

Step 3 The Boolean expression is this: *fs'p' + f'sp*

Step 4 This expression cannot be simplified since the two terms have no common variable.

Step 5 Here is the circuit.



EXERCISES 4-8

A *Give all steps in the solution of each problem.*

1. Revise Example **1** so that the machine offers at most one of *four* choices (add hot chocolate).

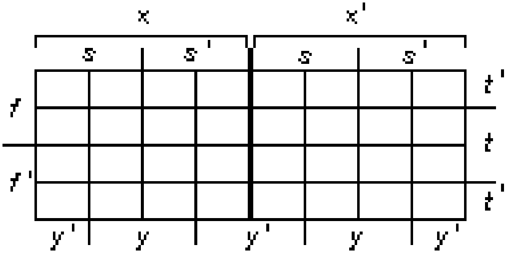
2. Revise Example **2** so that there are only two Counselors (eliminate Uncle Homer). Now design the missile launching system so that it is activated when at least two (any two) of the Council vote to fire.

3. Revise Example **2** so that the missiles are launched even without the President's vote if all three Counselors agree to fire.

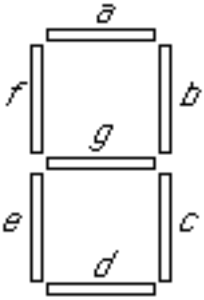
4. Revise Example **2** so that the missiles are launched only if the President and *at least two* of the Counselors agree to fire.

5. For the two-floor elevator of Example **3**, design the circuit that controls the door-opening mechanism. The inputs are the same as in Example **3**; replace output *m* with output *d* for opening the door when the elevator is at a floor.

C **6.** As in Example **3** design the circuitry for the "move" function but now for a three-floor elevator. There are three buttons: *f* = first floor button, *s* = second floor button, and *t* = third floor button. The present floor indicator requires two bits (*x* and *y*): 01 for first floor, 10 for second floor, and 11 for third floor. (Note that 00 is an impossible combination and should be omitted from the table.) For the output *m*, 1 means move to another floor; 0 means stay. On the next page is a five-variable Karnaugh map.



7. Most calculators, digital clocks, and watches use the “seven segment display” format. In this setup, as the diagram at the right shows, there are seven segments that can be lit in different combinations to form the numerals 0 through 9. For example, “1” is formed by lighting segments *b* and *c*; “2” consists of segments *a*, *b*, *g*, *e*, and *d*. “4” is composed of segments *b*, *c*, *f*, and *g*.



Design circuitry to run a seven-segment display for one digit. The input consists of a four-bit digit (where each bit is an input line). The outputs are *a*, *b*, *c*, *d*, *e*, *f*, and *g* of the seven segment diagram (1 = light the segment, 0 = do not light the segment). From a truth table, write and simplify *seven* Boolean expressions. Then draw the seven minimal circuits. Note: There are only *ten* rows of input in the table corresponding to the digits 0 (0000_{two} through 9 (1001_{two}).