AP® CALCULUS AB
2008 SCORING GUIDELINES

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

(a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

(c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

(d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

(a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = 8$ people per hour

$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

(b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4}\left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3)\right)$$

$= 155.25$ people

$2 : \begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

$3 : \begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

$3 : \begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[*Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t.*]

(d) $\int_0^3 r(t)\,dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

2A,

2 2 2 2 2

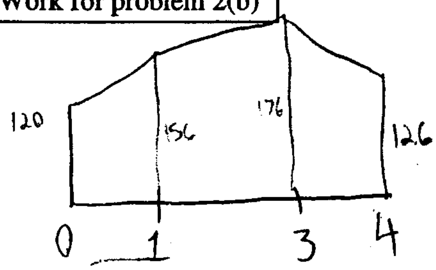
t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Work for problem 2(a)

$$L'(5.5) \approx \frac{L(7)-L(4)}{7-4} = \frac{150-126}{3} = \frac{24}{3} = 8$$

8 people in line per hour

Work for problem 2(b)



$$\frac{120+156}{2} + 156+176 + \frac{176+126}{2} =$$

↓

$138 + 156 + 176 + 151 = 621$

average people in line $= \dfrac{621}{4} = 155.25$ people

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Work for problem 2(c) Since $L(3) > L(1)$ and $L(3) > L(4)$, at some point between $t=1$ and $t=4$, the line must go from increasing to decreasing, and thus at some point $L'(t)=0$

Since $L(4) < L(3)$ and $L(4) < L7$, there must be a local minimum between $t=3$ and $t=7$, and thus another point where $L'(t)=0$

Since $L(7) > L(4)$ and $L(7) > L(8)$, there must be another local maximum between $t=4$ and $t=8$ and thus a point where $L'(t)=0$

There must be at least 3 points

Work for problem 2(d)

tickets sold $= \int_0^3 550te^{-\frac{t}{2}}\,dx = 972.784$

$\downarrow$

973

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2B1

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Work for problem 2(a)

$$L'(t) \approx \frac{L(7)-L(4) \text{ people}}{7-4 \text{ hour}} = \frac{150-126 \text{ people}}{3 \text{ hour}} = \frac{24 \text{ people}}{3 \text{ hour}} = \boxed{8 \frac{\text{People}}{\text{hour}}}$$

Work for problem 2(b)

$$\text{Average} = \frac{1}{b-a}\left(\frac{b-a}{2n}\right)\left(L(0) + 2L(1) + 2L(3) + L(4)\right)$$

$$= \frac{1}{6}\left(120 + 312 + 352 + 126\right) = \frac{910}{6} = \boxed{\frac{455}{3} \text{ People}}$$

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Continue problem 2 on page 7.

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Work for problem 2(c)

t	0	1	3	4	7	8	9
$L(t)$	120	156	176	126	150	80	0

$L'(t)$ must equal zero at least 3 times

because must equal zero whenever $L(t)$ changes from increasing to decreasing, or ~~from~~ decreasing to increasing, which we can only be sure of happening (based on the chart) between $t=3$ and $t=4$; between $t=4$ and $t=7$; and between $t=7$ and $t=8$.

Work for problem 2(d)

Tickets Sold $= \int_0^3 550te^{-t/2}\,dt \approx \boxed{973 \text{ tickets}}$

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2C1

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

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Work for problem 2(a)

$$\frac{f(7)-f(4)}{7-4} = \frac{150-126}{3} = 8 \ \frac{\text{people}}{\text{hr}}$$

Work for problem 2(b)

$$\frac{120+156}{2} + \frac{156+176}{4} + \frac{176+126}{2} = 372$$

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Continue problem 2 on page 7.

$\mathcal{X}c^{2}$

Work for problem 2(c)

2 times

Work for problem 2(d)

$r(t) = 550\, te^{-\frac{t}{2}}$ tickets/hour

$$\int_0^3 550te^{-\frac{t}{2}} = 274.966$$

≈ 275 tickets

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Question 2

Overview

This problem presented students with a table of data indicating the number of people $L(t)$ in line at a concert ticket office, sampled at seven times t during the 9 hours that tickets were being sold. (The question stated that $L(t)$ was twice differentiable.) Part (a) asked for an estimate for the rate of change of the number of people in line at a time that fell between the times sampled in the table. Students were to use data from the table to calculate an average rate of change to approximate this value. Part (b) asked for an estimate of the average number of people waiting in line during the first 4 hours and specified the use of a trapezoidal sum. Students needed to recognize that the computation of an average value involves a definite integral, approximate this integral with a trapezoidal sum, and then divide this total accumulation of people hours by 4 hours to obtain the average. Part (c) asked for the minimum number of solutions guaranteed for $L'(t) = 0$ during the 9 hours. Students were expected to recognize that a change in direction (increasing/decreasing) for a twice-differentiable function forces a value of 0 for its derivative. Part (d) provided the function $r(t) = 550te^{-t/2}$ tickets per hour as a model of the rate at which tickets were sold during the 9 hours and asked students to find the number of tickets sold in the first 3 hours, to the nearest whole number, using this model. Students needed to recognize that total tickets sold could be determined by a definite integral of the rate $r(t)$ at which tickets were sold.

Sample: 2A
Score: 9

The student earned all 9 points. In part (c) the student might have given a more complete justification for the existence of a local maximum on the interval $(1, 4)$. It would have been better if the student had used the word “graph” rather than “line” in the first paragraph. The response is more complete and uses better terminology in the second and third paragraphs, and the student gives the correct answer.

Sample: 2B
Score: 6

The student earned 6 points: 2 points in part (a), no points in part (b), 2 points in part (c), and 2 points in part (d). The student earned both points in part (a) with a correct estimate and correct units. In part (b) the student’s expression reflects subdivisions of $[0, 4]$ of equal length. The student did not earn any points. In part (c) the student earned the first point by considering $L(t)$ changing from increasing to decreasing. (The student goes on to consider $L(t)$ changing from decreasing to increasing, but this was not necessary to earn the first point.) The student did not earn the second point since it is not necessarily true that $L(t)$ changes from increasing to decreasing on the interval $[3, 4]$. The student earned the third point with the correct answer of 3. The student earned both points in part (d).

Question 2 (continued)

Sample: 2C
Score: 3

The student earned 3 points: 2 points in part (a), no points in part (b), no points in part (c), and 1 point in part (d). The student earned both points in part (a) with a correct estimate and correct units. The student's use of f rather than L was not penalized. The student did not earn points in part (b) because the given expression is not a valid trapezoidal sum. In part (c) the student did not earn the first point. As a result of the incorrect answer ("two times"), the student did not earn the other points in part (c). In part (d) the student earned the integrand point. Since the value of the integral is not correct, the student did not earn the second point, even though the limits on the integral are correct.